General Aptitude

Q. No. 1 – 5 Carry One Mark Each

1. Find the missing sequence in the letter series below:
   A, CD, GHI, ?, UVWXY
   (A) LMN  (B) MNO  (C) MNOP  (D) NOPQ
   Answer:  (C)
   Exp:
   \[
   \begin{array}{cccc}
   A & CD & GHI & MNOP \\
   B & E,F & JKL & UVWXY \\
   +1 & +2 & +3 & +4
   \end{array}
   \]

2. Choose the correct verb to fill in the blank below:
   Let us ____________.
   (A) Introvert  (B) alternate  (C) atheist  (D) altruist
   Answer:  (B)

3. Choose the most appropriate word from the options given below to complete the following sentence?
   If the athlete had wanted to come first in the race, he ____________ several hours every day.
   (A) Should practice  (B) Should have practised  (C) Practised  (D) Should be practicing
   Answer:  (B)
   Exp: For condition regarding something which already happened, should have practiced is the correct choice.

4. Choose the most suitable one word substitute for the following expression
   Connotation of a road or way
   (A) Pertinacious  (B) Viaticum  (C) Clandestine  (D) Ravenous
   Answer:  (B)

5. If \( x > y > 1 \), which of the following must be true?
   (i) \( \ln x > \ln y \)  (ii) \( e^x > e^y \)  (iii) \( y^x > x^y \)  (iv) \( \cos x > \cos y \)
   (A) (i) and (ii)  (B) (i) and (iii)  (C) (iii) and (iv)  (D) (ii) and (iv)
   Answer:  (B)
Q. No. 6 – 10 Carry Two Marks Each

6. From a circular sheet of paper of radius 30 cm, a sector of 10% area is removed. If the remaining part is used to make a conical surface, then the ratio of the radius and height of the cone is ________.

Answer: 2.06

Exp: 90% of area of sheet = Cross sectional area of cone

⇒ 0.9 × π × 30 × 30 = π × r₁ × 30
⇒ 27 cm = r₁

∴ Height of the cone = √30² – 27²
= 13.08 cm

Then r/h=27/13.08=2.06

7. In the following question, the first and the last sentence of the passage are in order and numbered 1 and 6. The rest of the passage is split into 4 parts and numbered as 2, 3, 4, and 5. These 4 parts are not arranged in proper order. Read the sentences and arrange them in a logical sequence to make a passage and choose the correct sequence from the given options.

(A) 2, 5, 3, 4  \hspace{1cm} (B) 5, 2, 4, 3  \hspace{1cm} (C) 3, 5, 4, 2  \hspace{1cm} (D) 4, 5, 2, 3

Answer: (B)

8. Ms. X will be in Bagdogra from 01/05/2014 to 20/05/2014 and from 22/05/2014 to 31/05/2014. On the morning of 21/05/2014, she will reach Kochi via Mumbai.

Which one of the statements below is logically valid and can be inferred from the above sentences?

(A) Ms. X will be in Kochi for one day, only in May
(B) Ms. X will be in Kochi for only one day in May
(C) Ms. X will be only in Kochi for one day in May
(D) Only Ms. X will be in Kochi for one day in May.

Answer: (B)
9. \[ \log \tan 1° + \log \tan 2° + \ldots \ldots + \log \tan 89° \text{ is } \ldots \ldots \]

- (A) 1
- (B) \( \frac{1}{\sqrt{2}} \)
- (C) 0
- (D) \(-1\)

**Answer:** (C)

**Exp:**
\[
\log \tan 1° + \log \tan 89° = \log (\tan 1° \times \tan 89°) \\
= \log (\tan 1° \times \cot 1°) \\
= \log 1 \\
= 0
\]

Using the same logic total sum is "0".

10. Ram and Shyam shared a secret and promised to each other that it would remain between them. Ram expressed himself in one of the following ways as given in the choices below. Identify the correct way as per standard English.

- (A) It would remain between you and me.
- (B) It would remain between I and you
- (C) It would remain between you and I
- (D) It would remain with me.

**Answer:** (A)

Electronics and Communication Engineering

Q. No. 1 – 25 Carry One Mark Each

1. A coaxial cable is made of two brass conductors. The spacing between the conductors is filled with Teflon \( (\varepsilon_r = 2.1, \tan \delta = 0) \). Which one of the following circuits can represent the lumped element model of a small piece of this cable having length \( \Delta z \)?

- (A)
- (B)
- (C)
- (D)

**Answer:** (B)
Exp: Loss tangent \( \tan \delta = 0 = \frac{\sigma}{\omega} \in \)
\[
\sigma = 0
\]

G \rightarrow \text{Conductivity of the dielectric material}

So, \( \sigma = 0 = G \)

2. The phase margin (in degrees) of the system \( G(s) = \frac{10}{s(s+10)} \) is _____.

Answer: 84.32

3. In the circuit shown, diodes \( D_1, D_2 \) and \( D_3 \) are ideal, and the inputs \( E_1, E_2 \) and \( E_3 \) are ‘0 V’ for logic ‘0’ and ‘10 V’ for logic ‘1’. What logic gate does the circuit represent?

(A) 3 input OR gate  
(B) 3 input NOR gate  
(C) 3 input AND gate  
(D) 3 input XOR gate

Answer: (C)

Exp: Case (i) : If any input is logic 0 (i.e., 0V) then corresponding diode is “ON” and due to ideal diode output voltage \( V_o = 0 \) as well as if there is any input logic 1 (i.e., 10V) corresponding diode will be OFF.

Case (ii) : If all the inputs are high (i.e., 10V) then all the diodes are R.B (OFF) and output voltage \( V_o = 10V \).

So, it is a positive logic 3-inputs AND gate.

4. In the circuit shown in the figure, the BJT has a current gain (\( \beta \)) of 50. For an emitter base voltage \( V_{EB} = 600 \text{ mV} \), the emitter collector voltage \( V_{EC} \) (in Volts) is ______.

Answer: 2.04
Exp: \[ V_{EB} = 0.7 \text{V} \]
\[ I_B = 0.0383 \text{mA} \]
\[ I_C = 1.916 \text{mA} \]
\[ V_{EC} = 3 - I_C R_C = 3 - (1.916 \times 0.5) = 2.04 \text{V} \]

5. The contour on the x-y plane, where the partial derivative of \( x^2 + y^2 \) with respect to \( y \) is equal to the partial derivative of \( 6y + 4x \) with respect to \( x \), is
(A) \( y = 2 \)  
(B) \( x = 2 \)  
(C) \( x + y = 4 \)  
(D) \( x - y = 0 \)

Answer: \( \text{(A)} \)

Exp: The partial derivative of \( x^2 + y^2 \) with respect to \( y \) is \( 0 + 2y \).
The partial derivative of \( 6y + 4x \) with respect \( x \) is \( 0 + 4 \).
Given that both are equal.
\[ 2y = 4 \Rightarrow y = 2 \]

6. For \( A = \begin{bmatrix} 1 & \tan x \\ -\tan x & 1 \end{bmatrix} \) the determinant of \( A^T A^{-1} \) is
(A) \( \sec^2 x \)  
(B) \( \cos 4x \)  
(C) \( 1 \)  
(D) \( 0 \)

Answer: \( \text{(C)} \)

Exp: \[ |A^T A| = |A|^2 |I| - 1 \]

7. In the circuit shown, the voltage \( V_x \) (in Volts) is______.

![Circuit Diagram](image)

Answer: 8

Exp:

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Apply KCL at point P
\[ V_x + \frac{V_x - 0.25V_x}{20} + 0.5V_x = 5 \]
\[ V_x \left(1 + \frac{0.75}{20} + 0.5\right) = 5 \]
\[ V_x \left(\frac{5}{8}\right) = 5 \Rightarrow V_x = 8V \]

8. Which one of the following 8085 microprocessor programs correctly calculates the product of two 8-bit numbers stored in registers B and C?

(A) MVI A, 00 H
JNZ LOOP
CMP C
LOOP DCR B
JNZ LOOP
HLT

(B) MVI A, 00H
CMP C
LOOP DCR B
JNZ LOOP
HLT

(C) MVI A, 00H
LOOP ADD C
DCRB
JNZ LOOP
HLT

(D) MVI A, 00H
LOOP ADD C
DCRB
JNZ LOOP
LOOP INR B
HLT

Answer: (C)

Exp: MVI A, 00H \leftarrow Load accumulator by 00H

Loop: ADDC \leftarrow Add the content of accumulator with content of C register and store result in accumulator.
This will continue till B register reaches to 004.

So, repetitive addition of a number as many times will give the product of these two numbers.

9. Consider the function \( g(t) = e^{-t} \sin(2\pi t) u(t) \) where \( u(t) \) is the unit step function. The area under \( g(t) \) is ________.

Answer: 0.155

Exp: \( g(t) = e^{-t} \sin(2\pi t).u(t) \)
Let \( y(t) = \sin(2\pi t).u(t) \)
then \( Y(s) = \frac{2\pi}{s^2 + (2\pi)^2} \)
10. In the circuit shown using an ideal opamp, the 3-dB cut-off frequency (in Hz) is _______.

\[ f_{3dB} = \frac{1}{2\pi RC} \]

Exp: 
\[ f_{3dB} = \frac{1}{2\pi \times 10 \times 10^3 \times 0.1 \times 10^{-6}} = 159.15 \text{Hz} \]

11. The modulation scheme commonly used for transmission from GSM mobile terminals is
(A) 4-QAM
(B) 16-PSK
(C) Walsh-Hadamard orthogonal codes
(D) Gaussian Minimum Shift Keying (GMSK)

Answer: (D)

12. Which one of the following processes is preferred to form the gate dielectric (SiO}_2 \text{) of MOSFETs?}
(A) Sputtering
(B) Molecular beam epitaxy
(C) Wet oxidation
(D) Dry oxidation

Answer: (D)
13. Consider the Bode plot shown in figure. Assume that all the poles and zeros are real valued.

The value of $f_H - f_L$ (in Hz) is ______.

Answer:  8970

Exp: \[
40 = \log_{10} \left( \frac{300}{f_L} \right) - \log_{10} \left( \frac{f_H}{900} \right)
\]

\[
\log_{10} \left( \frac{300}{f_L} \right) = 1
\]

\[
300 = 10f_L
\]

\[
f_L = 30Hz \quad \text{(i)}
\]

\[
f_H = 900 \times 10 = 9000 \quad \text{(ii)}
\]

\[
f_H - f_L = 9000 - 30 = 8970 \text{Hz}
\]

14. In the circuit shown, assume that diodes $D_1$ and $D_2$ are ideal. In the steady-state condition the average voltage $V_{ab}$ (in Volts) across the 0.5 μF capacitor is _____.

Answer:  100

15. The transfer function of a first order controller is given as $G_c(s) = \frac{K(s + a)}{s + b}$ where, $K$, $a$ and $b$ are positive real numbers. The condition for this controller to act as a phase lead compensator is

(A) $a < b$ \hspace{1cm} (B) $a > b$

(C) $K < ab$ \hspace{1cm} (D) $K > ab$

Answer:  (A)
Exp: For phase lead compensator

\[ s = -b \]
\[ s = -a \]
\[ a < b \]

16. A message signal \( m(t) = A_m \sin(2\pi f_m t) \) is used to modulate the phase of a carrier \( A_c \cos(2\pi f_c t) \) to get the modulated signal \( y(t) = A_c \cos(2\pi f_c t + m(t)) \). The bandwidth of \( y(t) \)
(A) depends on \( A_m \) but not on \( f_m \)
(B) depends on \( f_m \) but not on \( A_m \)
(C) depends on both \( A_m \) and \( f_m \)
(D) does not depend on \( A_m \) or \( f_m \)

Answer: (C)

Exp: 
\[ y(t) = A_c \cos \left[ 2\pi f_c t + m(t) \right] \]
\[ m(t) = A_m \sin \left( 2\pi f_m t \right) \]
Since \( y(t) \) is phase modulated signal,
\[ \phi(t) = 2\pi f_c t + m(t) \]
Bandwidth = \( 2 \left[ \Delta f + f_m \right] \)
\[ \Delta f = \frac{1}{2\pi} \frac{d}{dt} m(t) \]
\[ \Rightarrow \Delta f \text{ depends on } A_m \text{ as well as } f_m. \text{ Thus Bandwidth depends on both } A_m \text{ and } f_m. \]

17. The directivity of an antenna array can be increased by adding more antenna elements, as a larger number of elements
(A) improves the radiation efficiency
(B) increases the effective area of the antenna
(C) results in a better impedance matching
(D) allows more power to be transmitted by the antenna

Answer: (B)

Exp: 
\[ D = \frac{4\pi}{\lambda^2} A_e \]
\[ D \uparrow \rightarrow A_e \uparrow \]

18. For the circuit shown in the figure, the Thevenin equivalent voltage (in Volts) across terminals
a-b is _____.

![Circuit Diagram](image-url)
Answer: 10

Exp:

\[
\begin{align*}
\text{Apply nodal equation at point } P \\
V_{in} \left( \frac{1}{3} + \frac{1}{6} \right) &= \frac{12}{3} + 1 \\
V_{in} &= \frac{2}{5} \Rightarrow V_{in} = 10V
\end{align*}
\]

19. The impulse response of an LTI system can be obtained by
(A) differentiating the unit ramp response
(B) differentiating the unit step response
(C) integrating the unit ramp response
(D) integrating the unit step response

Answer: (B)

Exp: Let \( h(t) \) be the impulse response of the system

\[ y(t) \text{ is unit step response of the system} \]

\[ y(t) = \int_{-\infty}^{1} h(\tau) \, d\tau \]

If we need to get \( h(t) \), then we have to differentiate \( y(t) \).

Thus differentiating the unit-step response gives impulse response for LTI system.

20. Consider a four point moving average filter defined by the equation \( y[n] = \sum_{i=0}^{3} \alpha_i [n-i] \)

The condition on the filter coefficients that results in a null at zero frequency is
(A) \( \alpha_1 = \alpha_2 = 0; \alpha_0 = -\alpha_3 \)
(B) \( \alpha_1 = \alpha_2 = 1; \alpha_0 = -\alpha_3 \)
(C) \( \alpha_0 = \alpha_3 = 0; \alpha_1 = \alpha_2 \)
(D) \( \alpha_1 = \alpha_2 = 0; \alpha_0 = \alpha_3 \)

Answer: (A)

Exp: Given \( y[n] = \sum_{i=0}^{3} \alpha_i x(n-i) \)

\[ \Rightarrow y[n] = \alpha_0 x[n] + \alpha_1 x[n-1] + \alpha_2 x[n-2] + \alpha_3 x[n-3] \]

Getting a null at zero frequency implies that given filter can be high pass filter but it cannot be low pass filter.

High pass filter is possible if we have negative coefficients.

Let say, \( \alpha_1 = \alpha_2 = 0, \alpha_0 = -\alpha_3 \)
\[ y[n] = -\alpha_3 x[n] + \alpha_1 x[n - 3] \]

\[ H(z) = -\alpha_3 \left[ 1 - z^{-3} \right] \]

\[ \Rightarrow H(e^{j\Omega}) = -\alpha_3 \left[ 1 - e^{-j3\Omega} \right] \]

\[ = -\alpha_3 e^{-j3\Omega} \left[ \frac{e^{j\frac{3\Omega}{2}} - e^{-j\frac{3\Omega}{2}}}{2j} \right] \times 2j \]

\[ = -\alpha_3 2j \sin \left( \frac{3\Omega}{2} \right) e^{-j\frac{3\Omega}{2}} \]

\[ = -\alpha_3 2 \sin \frac{3\Omega}{2} e^{-j\frac{3\Omega}{2}} \]

\[ \Rightarrow H(e^{j\Omega}) \bigg|_{\Omega = 0} = 0 \]

In other cases it is not possible.

21. If \( C \) is a circle of radius \( r \) with centre \( z_0 \) in the complex \( z \)-plane and if \( n \) is a non-zero integer, then \( \oint_{C} \frac{dz}{(z-z_0)^{n+1}} \) equals

(A) \( 2\pi nj \)
(B) \( 0 \)
(C) \( \frac{n!}{2\pi} \)
(D) \( 2\pi n \)

**Answer:** (B)

**Exp:** By Cauchy’s Integral formula,

\[ \oint_{C} \frac{f(z)}{(z-z_0)^{n+1}} \, dz = \frac{2\pi i f^n(z_0)}{n!} \]

\[ \oint_{C} \frac{dz}{(z-z_0)^{n+1}} = \frac{2\pi i}{n!} \times 0 = 0 \]

\[ \therefore f(z) = 1 \]

\[ f^n(z) = 0 \text{ at any } z_0 \]

22. At very high frequencies, the peak output voltage \( V_0 \) (in Volts) is ______.

**Answer:** 0.5
Exp: For capacitor \( Z_C = \frac{1}{j\omega C} \)

Very high frequency means \( \omega \to \infty \Rightarrow Z_C \to 0 \)

So, all capacitors are replaced by short circuit.

![Circuit Diagram]

By voltage division, \( V_0 = \frac{V}{2} \)

\[ V_0 = \frac{1}{2} \cdot \sin \omega t = 0.5 \sin \omega t \]

Thus, Peak voltage = 0.5

23. If the base width in a bipolar junction transistor is doubled, which one of the following statements will be TRUE?
   (A) Current gain will increase
   (B) Unity gain frequency will increase
   (C) Emitter base junction capacitance will increase
   (D) Early voltage will increase

Answer: (D)

Exp: \( \text{W_B} \) doubled (increased) \( \Rightarrow \) early effect is still present but its effect less severe relative to previous \( \text{W_B} \). Slope \( I_C \text{Vs} V_{CE} \) decreases

24. The value of \( \sum_{n=0}^{\infty} n \left( \frac{1}{2} \right)^n \) is ____.

Answer: 2

Exp: Given that \( \sum_{n=0}^{\infty} n \left( \frac{1}{2} \right)^n \)

\[ = 0 + 1 \cdot \frac{1}{2} + 2 \left( \frac{1}{2} \right)^2 + 3 \left( \frac{1}{2} \right)^3 + 4 \left( \frac{1}{2} \right)^4 + \ldots \]

\[ = \frac{1}{2} \left[ 1 + 2 \left( \frac{1}{2} \right) + 3 \left( \frac{1}{2} \right)^2 + 4 \left( \frac{1}{2} \right)^3 + \ldots \right] \]

\[ = \frac{1}{2} \left[ 1 - \frac{1}{2} \right]^{-2} \quad \text{\( \because (1-x)^{-2} = 1 + 2x + 3x^2 + 4x^3 + \ldots \)} \]

\[ = \frac{1}{2} \left( \frac{1}{2} \right)^{-2} = \left( \frac{1}{2} \right)^{-1} = 2 \]
25. The circuit shown consists of J-K flip-flops, each with an active low asynchronous reset (R_d input). The counter corresponding to this circuit is

(A) a modulo-5 binary up counter  
(B) a modulo-6 binary down counter  
(C) a modulo-5 binary down counter  
(D) a modulo-6 binary up counter

Answer: (A)

Exp: Analysis:
1. Clock is taken from normal output and it is –ve edge triggering. So, it is UP-counter.
2. Input of the NAND-gate is taken from Q_2 and Q_o. So Q_2 = 1 and Q_o = 1.
3. To find the modulus

\[(Q_2, Q_1, Q_0) = (1, 0, 1)\]

So, it is MOD – 5 binary UP-counter.

Q. No. 26 – 55 Carry Two Marks Each

26. A 200 m long transmission line having parameters shown in the figure is terminated into a load R_L. The line is connected to a 400 V source having source resistance R_s through a switch which is closed at \(t = 0\). The transient response of the circuit at the input of the line (\(z = 0\)) is also drawn in the figure. The value of R_L (in \(\Omega\)) is ______

\[R_s = 150\Omega\]
\[R_o = 50\Omega\]
\[e_{r,eff} = 2.25\]
V_s = 400V

Answer: 30
Exp:

Given $V(t = 2\mu s, Z=0) = 62.5$

$62.5 = V(t = 0, z = 0) + V(t = 1, z = 0) + V(t = 2, z = 0)$

$62.5 = 100 + |\overline{R}(100)| + |\overline{R}S(100)|$

$|\overline{R}| = \frac{R_L - 50}{R_L + 50}$

$|S| = \frac{1}{2}$

So, $|\overline{R}| = \frac{-1}{4}$.

$R_L = 30 \Omega$

27. A coaxial capacitor of inner radius 1 mm and outer radius 5 mm has a capacitance per unit length of 172 pF/m. If the ratio of outer radius to inner is doubled, the capacitance per unit length (in pF/m) is.

Answer: 120.22

Exp: $C = \frac{2\pi \varepsilon}{\ln(b/a)}$

$C_1 = \frac{2\pi \varepsilon}{\ln(b_1/a_1)}$

$C_2 = \frac{\ln(b_2/a_2)}{\ln(b_1/a_1)}$

$172 \mu F = \frac{\ln(10/1)}{\ln(5)}$

$C_2 = \frac{\ln(5)}{\ln(10)} \times 172 \mu F$

$C_2 = 120.22 \mu F$

28. A universal logic gate can implement any Boolean function by connecting sufficient number of them appropriately. Three gates are shown.

$X \overrightarrow{\text{Y}} F_1 = X + Y$  $X \overrightarrow{\text{Y}} F_2 = X \cdot Y$  $X \overrightarrow{\text{Y}} F_3 = \overline{X} + Y$

Which one of the following statements is TRUE?
(A) Gate 1 is a universal gate.
(B) Gate 2 is a universal gate.
(C) Gate 3 is a universal gate.
(D) None of the gates shown is a universal gate.

Answer: (C)

29. The Newton-Raphson method is used to solve the equation \( f(x) = x^3 - 5x^2 + 6x - 8 = 0 \). Taking the initial guess as \( x = 5 \), the solution obtained at the end of the first iteration is _____.

Answer: 4.2903

Exp:

\[
f(x) = x^3 - 5x^2 + 6x - 8
\]

\[x_0 = 5\]

\[f'(x) = 3x^2 - 10x + 6\]

By Newton-Raphson method.

\[x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 5 - \frac{f(5)}{f'(5)}\]

\[= 5 - \frac{22}{31} = 5 - 0.7097 = 4.2903\]

30. A random binary wave \( y(t) \) is given by \( y(t) = \sum_{n=\infty}^{\infty} X_n p(t - nT - \phi) \).

where \( p(t) = u(t) - u(t - T) \), \( u(t) \) is the unit step function and \( \phi \) is an independent random variable with uniform distribution in \( [0, T] \). The sequence \( \{X_n\} \) consists of independent and identically distributed binary valued random variables with \( P\{X_n = +1\} = P\{X_n = -1\} = 0.5 \) for each \( n \).

The value of the auto correlation \( R_{yy}\left( \frac{3T}{4} \right) \) equals _______.

Answer: 0.25

Exp:

\[y(t) = \sum_{n=-\infty}^{\infty} X_n p(t - nT - \phi)\]

\[R_{yy}(\tau) = \mathbb{E} \left[ y(t) y(t - \tau) \right]\]

Derivation of above autocorrelation function can be found in any book dealing with random process.

[B.P. Lathi, Simon, Haykin, Schaum series].

\[R_{yy}\left( \frac{3T}{4} \right) = \left[ 1 - \frac{3\pi}{4} \right]\]

\[= \frac{1}{4} = 0.25\]
31. A three bit pseudo random number generator is shown. Initially the value of output \( Y = Y_2 Y_1 Y_0 \) is set to 111. The value of output \( Y \) after three clock cycles is

\[
Y_2 \, Y_1 \, Y_0 = 000, 001, 010, 100
\]

Answer: (D)

Exp:

\[
\begin{array}{c|c|c|c|c|c|c}
D_2 (Q_2 \oplus Q_0) & D_1 (Q_1) & D_0 (Q_0) & Y_2 (Q_2) & Y_1 (Q_1) & Y_0 (Q_0) \\
\hline
- & - & - & 1 & 1 & 1 \\
1^{st} \text{clock} & 0 & 1 & 1 & 0 & 1 & 1 \\
2^{nd} \text{clock} & 0 & 0 & 1 & 0 & 0 & 1 \\
3^{rd} \text{clock} & 1 & 0 & 0 & 1 & 0 & 0 \\
\end{array}
\]

After three clock pulses output \( Y_2 Y_1 Y_0 = 100 \)

32. In the circuit shown, assume that the opamp is ideal. If the gain (\( V_o/V_{in} \)) is –12, the value of \( R \) (in k\( \Omega \)) is ___

\[
R = 10 \, \text{k}\Omega
\]

Answer: 1

Exp:
Apply nodal analysis at inverting terminal
\[
\frac{V_{\text{in}}}{10k} = 0 - \frac{V_x}{10k} \Rightarrow V_{\text{in}} = -V_x \quad (1)
\]

Again apply nodal analysis at node \( V_x \)
\[
\frac{0 - V_x}{10k} = \frac{V_x - 0}{R} + \frac{V_x - V_0}{10k} \quad (2)
\]

Put the value of \( V_x \) from equation (1) in equation (2) we get
\[ R = 1k\Omega \]

33. Two sequences \( x_1[n] \) and \( x_2[n] \) have the same energy. Suppose \( x_1[n] = a \cdot 0.5^n u[n] \), where \( a \) is a positive real number and \( u[n] \) is the unit step sequence. Assume
\[
x_2[n] = \begin{cases} \sqrt{1.5} & \text{for } n = 0, 1 \\ 0 & \text{otherwise} \end{cases}
\]

Then the value of \( a \) is _____.
Answer: 1.5

Exp: \( x_1[n] = \alpha(0.5)^n u[n] \)

Energy of \( x_1[n] = \sum_{n=0}^{\infty} x_1^2[n] = \sum_{n=0}^{\infty} \alpha^2 (0.5)^{2n} = \alpha^2 \sum_{n=0}^{\infty} (0.25)^n = \alpha^2 \left[ \frac{1}{1-0.25} \right] = \alpha^2 \left[ \frac{4}{3} \right] \)

Energy of \( x_2[n] = \left( \sqrt{1.5} \right)^2 + \left( \sqrt{1.5} \right)^2 = 3. \)

\( \therefore \) The energy of two sequences is same,
\[ \alpha^2 \left( \frac{4}{3} \right) = 3 \Rightarrow \alpha^2 = \frac{9}{4} \]
\[ \therefore \alpha = \frac{3}{2} = 1.5 \]

34. The ABCD parameters of the following 2-port network are
(A) \[
\begin{bmatrix}
3.5 + j2 & 20.5 \\
20.5 & 3.5 - j2
\end{bmatrix}
\]
(B) \[
\begin{bmatrix}
3.5 + j2 & 0.5 \\
0.5 & 3.5 - j2
\end{bmatrix}
\]
(C) \[
\begin{bmatrix}
10 & 2 + j0 \\
2 + j0 & 10
\end{bmatrix}
\]
(D) \[
\begin{bmatrix}
7 + j4 & 0.5 \\
30.5 & 7 - j4
\end{bmatrix}
\]

Answer:  (B)

Exp:  For the standard ‘T’ network, obtain the Z-matrix first and then convert it into T-matrix

\[
Z = \begin{bmatrix}
7 + j4 & 2 \\
2 & 7 - j4
\end{bmatrix}
\]

\[
\Delta Z = \left[ (7 + j4)(7 - j4) \right] - 4
\]

\[= 49 + 16 - 4 = 61\]

\[A = \frac{Z_{21}}{Z_{11}} = \frac{7 + j4}{2} = 3.5 + j2\]

\[B = \frac{\Delta Z}{Z_{21}} = \frac{61}{2} = 30.5\]

\[C = \frac{1}{Z_{21}} = \frac{1}{2} = 0.5\]

\[D = \frac{1}{Z_{11}} = \frac{1}{2} = 0.5\]

\[T = \begin{bmatrix}
3.5 + j2 & 30.5 \\
0.5 & 3.5 - j2
\end{bmatrix}
\]

35.  A network is described by the state model as

\[
\dot{x}_1 = 2x_1 - x_2 + 3u
\]

\[
\dot{x}_2 = -4x_2 - u
\]

\[y = 3x_1 - 2x_2\]

The transfer function \(H(s)\) is

\[H(s) = C(sI - A)^{-1}B\]

(A) \[
\frac{11s + 35}{(s - 2)(s + 4)}
\]

(B) \[
\frac{11s - 35}{(s - 2)(s + 4)}
\]

(C) \[
\frac{11s + 38}{(s - 2)(s + 4)}
\]

(D) \[
\frac{11s - 38}{(s - 2)(s + 4)}
\]

Answer:  (A)

Exp:  

\[A = \begin{bmatrix}
2 & -1 \\
0 & -4
\end{bmatrix}
\]

\[B = \begin{bmatrix}
3 \\
-1
\end{bmatrix}
\]

\[C = \begin{bmatrix}
3 & -2
\end{bmatrix}
\]

\[H(s) = C(sI - A)^{-1}B\]
\[ = \begin{bmatrix} 3 & -2 \\ -2 & 0 \end{bmatrix} \begin{bmatrix} s-2 & 1 \\ 0 & s+4 \end{bmatrix}^{-1} \begin{bmatrix} 3 \\ -1 \end{bmatrix} \]

\[ = \begin{bmatrix} 3 & -2 \\ -2 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ \frac{1}{s-2(s+4)} \end{bmatrix} \begin{bmatrix} s+4 & -1 \\ 0 & s-2 \end{bmatrix}^{-1} \begin{bmatrix} 3 \\ -1 \end{bmatrix} \]

\[ = \begin{bmatrix} 3 & -2 \\ -2 & 0 \end{bmatrix} \frac{1}{s^2+2s-8} \begin{bmatrix} 3s+12+1 \\ -s+2 \end{bmatrix} \]

\[ = \frac{95+39+25-4}{(s-2)(s+4)} \]

\[ = \frac{11s+35}{(s-2)(s+4)} \]

36. In the circuit shown, the current \( I \) flowing through the 50 \( \Omega \) resistor will be zero if the value of capacitor \( C \) (in \( \mu \text{F} \)) is_____.

Answer: 20

Exp: Convert the network into phasor domain

If \( I = 0 \), \( V_p = V_i \),
\[
V_i \left( \frac{1}{j5} + \frac{1}{j(5-\frac{1}{500C})} \right) = 0
\]

\[
\Rightarrow \frac{1}{j5} = \frac{-1}{j(5-\frac{1}{500C})} = 0
\]

\[
\frac{1}{5} = \frac{1}{5000C - 5}
\]

\[
\Rightarrow 5 = \frac{1}{5000C} - 5
\]

\[
\Rightarrow \frac{1}{5000C} = 10
\]

\[
\Rightarrow C = \frac{1}{5 \times 10^3} = 20 \mu F
\]

37. A realization of a stable discrete time system is shown in figure. If the system is excited by a unit step sequence input \( x[n] \), the response \( y[n] \) is

\( x[n] \)

\( + \)

\( \frac{1}{5} \)

\( - \frac{5}{3} \)

\( z^{-1} \)

\( + \)

\( 1 \)

\( - \frac{2}{9} \)

\( \frac{5}{3} \)

\( z^{-1} \)

\( + \)

\( y[n] \)

(A) \( 4 \left( -1 \right)^n u(n) - 5 \left( -\frac{2}{3} \right)^n u[n] \)

(B) \( 5 \left( -\frac{2}{3} \right)^n u(n) - 3 \left( -\frac{1}{3} \right)^n u[n] \)

(C) \( 5 \left( \frac{1}{3} \right)^n u(n) - 5 \left( \frac{2}{3} \right)^n u[n] \)

(D) \( 5 \left( \frac{2}{3} \right)^n u(n) - 5 \left( \frac{1}{3} \right)^n u[n] \)

Answer: (C)

Exp: \( x[n] \) v[n]
From the graph
\[ v[n] = x[n] + v[n - 1] - \frac{2}{9} v[n - 2] \]
\[ y[n] = -\frac{5}{3} v[n - 1] + \frac{5}{3} v[n - 2] \]
\[ V(z) = \left[ 1 - z^{-1} + \frac{2}{9} z^{-2} \right] = X(z) \]
\[ \Rightarrow \frac{V(z)}{X(z)} = \frac{1}{1 - z^{-1} + \frac{2}{9} z^{-2}} \rightarrow (1) \]
\[ \frac{Y(z)}{V(z)} = -\frac{5}{3} z^{-1} + \frac{5}{3} z^{-2} \rightarrow (2) \]
Multiplying (1) and (2) we get
\[ \frac{Y(z)}{X(z)} = \frac{-\frac{5}{3} z^{-1} \left[ 1 - z^{-1} \right]}{1 - z^{-1} + \frac{2}{9} z^{-2}} \]
For unit step response, \[ X(z) = \frac{1}{1 - z^{-1}} \]
\[ \Rightarrow Y(z) = \frac{-\frac{5}{3} z^{-1}}{1 - z^{-1} + \frac{2}{9} z^{-2}} \]
\[ = \frac{A}{1 - \frac{1}{3} z^{-1}} + \frac{B}{1 - \frac{2}{3} z^{-1}} \]
On solving,
\[ A = 5 \ ; \ B = -5 \]
\[ \Rightarrow y[n] = 5 \left( \frac{1}{3} \right)^n u[n] - 5 \left( \frac{2}{3} \right)^n u[n] \]

38. The complex envelope of the bandpass signal \( x(t) = \sqrt{\frac{\sin(\pi t/5)}{\pi/5}} \sin \left( \pi t - \frac{\pi}{4} \right) \), centered about \( f = \frac{1}{2} \) Hz, is

(A) \( \left( \frac{\sin(\pi t/5)}{\pi/5} \right) e^{j\pi t} \)

(B) \( \frac{\sin(\pi t/5)}{\pi/5} e^{-j\pi t} \)

(C) \( \sqrt{\frac{\sin(\pi t/5)}{\pi/5}} e^{j\pi t} \)

(D) \( \sqrt{\frac{\sin(\pi t/5)}{\pi/5}} e^{-j\pi t} \)

Answer: (C)
Exp: \[ x(t) = -\sqrt{2} \left( \frac{\sin(\pi t/5)}{\pi t/5} \right) \sin \left( \frac{\pi t}{4} \right) \]

we can write above expression as

\[ x(t) = -\sqrt{2} \left( \frac{\sin(\pi t/5)}{\pi t/5} \right) \left[ \cos \left( \frac{\pi t}{4} \right) - \sin \left( \frac{\pi t}{4} \right) \cos \pi t \right] \]

\[ x(t) = \frac{\sin(\pi t/5)}{\pi t/5} \cos \pi t - \frac{\sin(\pi t/5)}{\pi t/5} \sin(\pi t) \]

Also

\[ x(t) = x_c(t) \cos 2\pi f t - x_s(t) \sin (2\pi f t) \]

[Low pass representation of Bandpass signals]

\[ x_c(t) = \frac{\sin(\pi t/5)}{\pi t/5}, \quad x_s(t) = \frac{\sin(\pi t/5)}{\pi t/5} \]

\[ x_{cs}(t) \] is the complex envelope of \( x(t) \)

\[ x_{cs}(t) = x_c(t) + jx_s(t) \]

\[ \frac{\sin(\pi t/5)}{\pi t/5} [1 + j] = \frac{\sqrt{2} \sin(\pi t/5)}{\pi t/5} e^{j\pi/4} \]

39. In the circuit shown, assume that the diodes \( D_1 \) and \( D_2 \) are ideal. The average value of voltage \( V_{ab} \) (in Volts), across terminals ‘a’ and ‘b’ is ________.

\[ \text{Answer: 5} \]

Exp: Input is positive half-cycle \( (D_1 = \text{ON}, D_2 = \text{OFF}) \)

\[ V_{ab} = \frac{6\pi}{3} \sin \omega t = 2\pi \sin \omega t \]

Input is Negative half-cycle \( (D_1 = \text{OFF}, D_2 = \text{ON}) \)

\[ V_{ab} = \frac{6\pi}{2} \sin \omega t = 3\pi \sin \omega t \]

Average of \( V_{ab} \) \[ \frac{2\pi}{\pi} + \frac{3\pi}{\pi} = 5 \text{ volts} \]
40. Consider the differential equation
\[ \frac{d^2 x(t)}{dt^2} + 3 \frac{dx(t)}{dt} + 2x(t) = 0. \]
Given \( x(0) = 20 \) and \( x(1) = \frac{10}{e} \), where \( e = 2.71 \), the value of \( x(2) \) is ____________

**Answer:** 0.8556

**Exp:**
Given

\[ \frac{d^2 x(t)}{dt^2} + 3 \frac{dx(t)}{dt} + 2x(t) = 0 \]

\( x(0) = 20 \)
\( x(1) = \frac{10}{e} \)
\( x(2) = - - - - - \)

Auxiliary equation is
\[ m^2 + 3m + 2 = 0 \]
\[ m = -1, -2 \]

Complementary solution
\[ x_c = c_1 e^{-t} + c_2 e^{-2t} \]

Particular solution \( x_p = 0 \)

Finally \( x = x_c + x_p = c_1 e^{-t} + c_2 e^{-2t} \)
\( x(0) = 20 \Rightarrow 20 = c_1 + c_2 \Rightarrow (a) \)
\( x(1) = \frac{10}{e} \Rightarrow \frac{10}{e} = c_1 e^{-1} + c_2 e^{-2} \)

\Rightarrow \( 10 = c_1 + c_2 e^{-1} \rightarrow (b) \)

From (a) \( c_1 = 20 - c_2 \)

Now \( 10 = (20 - c_2) + c_2 e^{-1} \)

\[ 10 = c_2 \left( e^{-1} - 1 \right) + 20 \]

\[ c_2 = \frac{10 - 20}{e^{-1} - 1} = \frac{10}{1 - e^{-1}} = \frac{10e}{e - 1} \]

\Rightarrow \( c_1 = 20 - \frac{10e}{e - 1} = \frac{20e - 20 - 10e}{e - 1} = \frac{10e - 20}{e - 1} \)

\( x(1) = \frac{10e - 20}{e - 1} e^{-t} + \frac{10e}{e - 1} e^{-2t} \)
\( x(2) = \left( \frac{10e - 20}{e - 1} \right) e^{-2} + \left( \frac{10e}{e - 1} \right) e^{-4} = 0.8556 \)

41. Let \( \tilde{x}[n] = 1 + \cos \left( \frac{\pi n}{8} \right) \) be periodic signal with period 16. Its DFS coefficient are defined

by \( \alpha_k = \frac{1}{16} \sum_{n=0}^{15} \tilde{x}[n] \exp \left( -j \frac{\pi}{8} kn \right) \) for all k. The value of the coefficient \( \alpha_{31} \) is ____________

**Answer:** 0.5
Exp: \( x[n] = 1 + \cos\left(\frac{\pi}{8}n\right) \)

N=16

\( x[n] = 1 + \frac{1}{2}e^{\frac{2\pi n}{16}} + \frac{1}{2}e^{-\frac{2\pi n}{16}} \)

\( a_{-1} = \frac{1}{2}, a_1 = \frac{1}{2}, a_0 = 1 \)

\( a_1 = a_{16+16} \Rightarrow a_{-1} = a_{15} = \frac{1}{2} \)

\( \Rightarrow a_0 = 1, a_1 = \frac{1}{2}, a_2 \text{ to } a_{14} = 0, a_{15} = \frac{1}{2} \)

DFS coefficients are also periodic with period 16.

\( a_{31} = a_{16+15} \)

\( a_{31} = a_{15} \)

\( \Rightarrow a_{31} = \frac{1}{2} \)

42. A fair die with faces \{1, 2, 3, 4, 5, 6\} is thrown repeatedly till ‘3’ is observed for the first time. Let \( X \) denote the number of times the die is thrown. The expected value of \( X \) is __________.

Answer: 6

Exp: Probability of getting 3 = \( \frac{1}{6} \)

Probability of not getting 3 = \( 1 - \frac{1}{6} = \frac{5}{6} \)

If dice thrown repeatedly till first 3 observed first time then

\[
E(x) = \frac{1}{6} + 2 \left( \frac{5}{6} \times \frac{1}{6} \right) + 3 \left( \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} \right) + \ldots
\]

\[
= \frac{1}{6} \left[ 1 + 2 \left( \frac{5}{6} \right) + 3 \left( \frac{5}{6} \right)^2 + \ldots \right]
\]

\[
= \frac{1}{6} \left( 1 - \left( \frac{5}{6} \right) \right)^{-2}
\]

\[
= \frac{1}{6} \times 36 = 6
\]
43. The electric field profile in the depletion region of a \( p-n \) junction in equilibrium is shown in the figure. Which one of the following statements is NOT TRUE?

(A) The left side of the junction is \( n \)-type and the right side is \( p \)-type
(B) Both the \( n \)-type and \( p \)-type depletion regions are uniformly doped
(C) The potential difference across the depletion region is 700 mV
(D) If the \( p \)-type region has a doping concentration of \( 10^{15} \, \text{cm}^{-3} \), then the doping concentration in the \( n \)-type region will be \( 10^{16} \, \text{cm}^{-3} \)

**Answer:** (C)

**Exp:**

Built in potential

\[
\psi_0 = \frac{1}{2} \times \left( 10^6 \, \text{V/m} \right) \times \left( 1.1 \times 10^{-4} \, \text{m} \right)
\]

= 0.55 volts

But in Question (option C) is given as 700 mV.

44. A vector field \( \mathbf{D} = 2\rho^2 \, \mathbf{\hat{a}}_{\rho} + z \, \mathbf{\hat{a}}_z \) exists inside a cylindrical region enclosed by the surfaces \( \rho = 1 \), \( z = 0 \) and \( z = 5 \). Let \( S \) be the surface bounding this cylindrical region. The surface integral of this field on \( S \) is _____.

**Answer:** 78.53

**Exp:**

\[
\oint_S \mathbf{D} \cdot d\mathbf{s} = \int_V (\nabla \cdot \mathbf{D}) \, dv
\]

\[
\nabla \cdot \mathbf{D} = \frac{1}{\rho} \frac{\partial}{\partial \rho} \left( \rho D_\rho \right) + \frac{1}{\rho} \frac{\partial D_\phi}{\partial \phi} + \frac{\partial D_z}{\partial z}
\]

\[
= \frac{1}{\rho} \frac{\partial}{\partial \rho} \left( \rho 2\rho^2 \right) + 0 + 1
\]

\[
= \frac{1}{\rho} 2(3)\rho^2 + 1
\]

\[
= 6\rho + 1
\]

\[
\oint_S \mathbf{D} \cdot d\mathbf{s} = \int_V (6\rho + 1) \, dv
\]

\[
= \int_0^1 \int_0^5 \int_0^\infty (6\rho + 1) \, \rho \, d\rho \, dz \, dv
\]

\[
= \frac{78.53}{4} \pi
\]

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\[
\int (\nabla \cdot D) \, dv = \int_{\rho=0}^{1} \int_{\phi=0}^{2\pi} \int_{z=0}^{5} (\rho + 1) \rho \, d\rho \, d\phi \, dz
\]
\[
= \left( \frac{\rho^3}{3} + \frac{\rho^2}{2} \right) \bigg|_{\rho=0}^{1} \left( 2\pi \right) (5)
\]
\[
= \left( 2 + \frac{1}{2} \right) 10\pi
\]
\[
\int (\nabla \cdot D) \, dv = 78.53
\]

45. An npn BJT having reverse saturation current \( I_s = 10^{-15} \, \text{A} \) is biased in the forward active region with \( V_{BE} = 700 \, \text{mV} \). The thermal voltage \( (V_T) \) is 25 mV and the current gain \( (\beta) \) may vary from 50 to 150 due to manufacturing variations. The maximum emitter current (in \( \mu\text{A} \)) is _____.

**Answer:** 1475

**Exp:**
\[
I_E = \frac{I_C}{\beta} = \frac{I_s}{\beta} e^{V_{BE}/V_T}
\]
\[
I_E = (\beta + 1) I_B
\]
\[
= \frac{\beta + 1}{\beta} I_S e^{V_{BE}/V_T}
\]
\[
= (1.02) \left( 10^{-9} \times 10^{-8} \right) e^{700 \times 10^{-3}/25 \times 10^{-3}}
\]
\[
= 1475 \, \mu\text{A}
\]

46. Consider the 3 m long lossless air-filled transmission line shown in the figure. It has a characteristic impedance of 120\( \pi \) \( \Omega \), is terminated by a short circuit, and is excited with a frequency of 37.5 MHz. What is the nature of the input impedance \( (Z_{in}) \)?

(A) Open (B) Short (C) Inductive (D) Capacitive

**Answer:** (D)

**Exp:**
\[
Z_{in} = JZ_0 \tan \beta l
\]
\[
\beta l = \frac{2\pi}{\lambda} l = \frac{3 \times 10^4}{37.5 \times 10^6} = 8
\]
\[
\Rightarrow l = \frac{3\pi}{4}
\]
Short circuited line
47. The current in an enhancement mode NMOS transistor biased in saturation mode was measured to be 1 mA at a drain-source voltage of 5 V. When the drain-source voltage was increased to 6 V while keeping gate-source voltage same, the drain current increased to 1.02 mA. Assume that drain to source saturation voltages is much smaller than the applied drain-source voltage. The channel length modulation parameter $\lambda$ (in V$^{-1}$) is __________.

**Answer:** 0.022

**Exp:**

$V_{DSat} \ll V_{DS}$

$V_{DS} = 5V$

$V_{DS} = 6V$

$I_D = k (V_{DS} - V_T)^2 (1 + \lambda V_{DS})$

$10^{-3} = k' (1 + 5\lambda)$

$1.02 \times 10^{-3} = k' (1 + 6\lambda)$

$1.02 = \frac{1 + 6\lambda}{1 + 5\lambda}$

$0.02 = 0.9\lambda$

$\lambda = 0.022 V^{-1}$

48. For the system shown in figure, $s = -2.75$ lies on the root locus if $K$ is ______.

**Answer:** 0.3

**Exp:**

$G(s)H(s) = \frac{10k(s+3)}{(s+2)}$

$= \frac{k'(s+3)}{(s+2)}$

For $k'$

$k' = \frac{0.75}{0.25} = 3$

$10k = 3$

$k = 0.3$
49. An SR latch is implemented using TTL gates as shown in the figure. The set and reset pulse inputs are provided using the push-button switches. It is observed that the circuit fails to work as desired. The SR latch can be made functional by changing

(A) NOR gates to NAND gates
(B) inverters to buffers
(C) NOR gates to NAND gates and inverters to buffers
(D) 5 V to ground

Answer: (D)

50. The variance of the random variable X with probability density function \( f(x) = \frac{1}{2} |x| e^{\frac{-|x|}{2}} \) is ______

Answer: 6

Exp: Given that \( f(x) = \frac{1}{2} |x| e^{\frac{-|x|}{2}} \) is probability density function of random variable X.

\[ V(x) = E(x^2) - [E(x)]^2 \]

\[ E(x) = \int_{-\infty}^{\infty} x f(x) \, dx = \int_{-\infty}^{\infty} \frac{1}{2} |x| e^{\frac{-|x|}{2}} \, dx \]

\[ = 0 \quad (\because \text{the function is odd}) \]

\[ E(x)^2 = \int_{-\infty}^{\infty} x^2 f(x) \, dx \]

\[ = \int_{-\infty}^{\infty} x^2 \cdot \frac{1}{2} |x| e^{\frac{-|x|}{2}} \, dx \]

\[ = \frac{2}{3} \int_{0}^{\infty} x^3 e^{-x} \, dx \quad (\because \text{function is even}) \]

\[ = 3! = 6 \]

51. Consider a continuous-time signal defined as

\[ x(t) = \left( \frac{\sin(at/2)}{\pi t/2} \right) * \sum_{n=0}^{\infty} \delta(t - 10n) \]

Where \( * \) denotes the convolution operation and \( t \) is in seconds. The Nyquist sampling rate (in samples / sec) for \( x(t) \) is ______.

Answer: 0.4

Exp: \[ x(t) = \frac{\sin\left( \frac{\pi t}{2} \right)}{\left( \frac{\pi t}{2} \right)} * \sum_{n=0}^{\infty} \delta(t - 10n) \]

Convolution in time domain becomes multiplication in frequency domain.
\[
\frac{1}{10} \sum_{n=-\infty}^{\infty} \delta(f - kf_n)
\]

\[F_s = \frac{1}{T_s} = 0.1\]

\[
\frac{\sin(\pi t / 2)}{(\pi t / 2)}
\]

\[
\sum_{n=-\infty}^{\infty} \delta(t - 10n)
\]

Multiplication in frequency domain will result maximum frequency is 0.2.

Thus Nyquist rate = 0.4 samples/sec

52. In the circuit shown, the both the enhancement mode NMOS transistors have the following characteristics: \(k_n = \mu_n C_{ox} (W/L) = 1\ mA/V^2\); \(V_{TN} = 1\ V\). Assume that the channel length modulation parameter \(\lambda\) is zero and body is shorted to source. The minimum supply voltage \(V_{DD}\) (in volts) needed to ensure that transistor \(M_1\) operates in saturation mode of operation is _____.

**Answer:** 3

**Exp:** Lower transistor (\(M_1\)) to work in saturation

\(V_{DS1} \geq V_{GSI} - V_s\)

So, for minimum \(V_{DD}\)

\(V_{DS1} = V_{GSI} - V_s\)

\(V_{DS1} = 2 - 1 = 1\ V\)

\(V_{DS1} = V_{D1} - V_{S1}\)

\(1\ V = V_{D1} \times 0\)

\(\therefore V_{D1} = 1\ V\)

and \(I_{D1} = K'(V_{GSI} - V_s)^2\)

\(I_{D1} = \frac{1\ mA}{V^2} \times (2 - 1)^2 = 1\ mA\)
Now transistor $M_2$, $V_{DG} = 0V$
So, it will work into saturation region and same current will flow

$\begin{align*}
I_{D2} = I_{D1} &= K' (V_{GS2} - V_t)^2 \\
1mA &= 1mA/V^2 \times (V_{DD} - 1 - 1)^2 \quad (\because V_{S2} = V_{D1}) \\
\therefore V_{DD} &= 3V
\end{align*}$

53. The position control of a DC servo-motor is given in the figure. The values of the parameters are $K_T = 1 \text{ N-m/A}$, $R_a = 1 \Omega$, $L_a = 0.1 \text{H}$, $J = 5 \text{ kg-m}^2$, $B = 1 \text{ N-m (rad/sec)}$ and $K_b = 1 \text{ V/(rad/sec)}$. The steady-state position response (in radians) due to unit impulse disturbance torque $T_d$ is _________.

\[ \theta(s) = \frac{1}{s(1s + B) + \frac{K_b K_T}{R_a + L_a s}} \]

Answer: $-0.5$

Exp: $T_d(s) = 1$

\[ \theta(s) = \lim_{s \to 0} \frac{s}{s(1s + B) + \frac{K_b K_T}{R_a + L_a s}} \]

Steady State Value is $\lim_{s \to 0} \theta(s) = -0.5$

54. The characteristic equation of an LTI system is given by $F(s) = s^5 + 2s^4 + 3s^3 + 6s^2 - 4s - 8 = 0$. The number of roots that lie strictly in the left half $s$-plane is _________.

\[ s^n \begin{array}{ccc} 1 & 3 & -4 \\ 2 & 6 & -8 \\ 8 & 12 & 0 \\ 3 & -8 & 0 \rightarrow as \epsilon \rightarrow 0 \\ -9.33 & 0 & 0 \\ -8 \end{array} \]

\[ 2s^4 + 6s^2 - 8 = 0 \]

Let $x = s^2$, then

\[ 2x^2 + 6x - 8 \Rightarrow x = 1, -4 \]

\[ s^2 = 1, -4 \Rightarrow s = \pm 1, \pm 2j \]
Number of roots lies on RHS: \( s = 1 \)
There are only two poles left on LHS.

55. Suppose \( x[n] \) is an absolutely summable discrete-time signal. Its \( z \)-transform is a rational function with two poles and two zeroes. The poles are at \( z = \pm 2j \). Which one of the following statements is TRUE for the signal \( x[n] \)?

(A) It is a finite duration signal.
(B) It is a causal signal.
(C) It is a non-causal signal.
(D) It is a periodic signal

**Answer:** (C)

**Exp:** Since \( x[n] \) is absolutely summable thus its ROC must include unit circle.

Thus ROC must be inside the circling radius 2. \( x[n] \) must be a non-causal signal.