General Aptitude

Q. No. 1 – 5 Carry One Mark Each

1. Five teams have to compete in a league, with every team playing every other team exactly once before going to the next round. How many matches will have to be held to complete the league round of matches?
   (A) 20 (B) 10 (C) 8 (D) 5
   \[ \text{Answer: (B)} \]
   \[ \text{Exp: For a match to be played, we need 2 teams} \]
   \[ \text{No. of matches} = \text{No. of ways of selections 2 teams out of 5} = ^5C_2 = 10 \]

2. Fill in the blank with the correct idiom/phrase.
   That boy from the town was a _____ in the sleepy village.
   (A) dog out of herd (B) sheep from the heap
   (C) fish out of water (D) bird from the flock
   \[ \text{Answer: (C)} \]
   \[ \text{Exp: From the statement, it appears that boy found it tough to adapt to a very different situation.} \]

3. Choose the statement where underlined word is used correctly.
   (A) When the teacher eludes to different authors, he is being elusive.
   (B) When the thief keeps eluding the police, he is being elusive.
   (C) Matters that are difficult to understand, indentify or remember are allusive.
   (D) Mirages can be allusive, but a better way to express them is illusory.
   \[ \text{Ans: (B)} \]
   \[ \text{Exp: Elusive: Difficult to answer.} \]

4. Tanya is older than Eric. Cliff is older than Tanya. Eric is older than Cliff.
   If the first two statements are true, then the third statement is:
   (A) True (B) False (C) Uncertain (D) Data insufficient
   \[ \text{Ans: (B)} \]

5. Choose the appropriate word/phrase, out of the four options given below, to complete the following sentence:
   Apparent lifelessness _________ dormant life.
   (A) harbours (B) leads to (C) supports (D) affects
   \[ \text{Answer: (A)} \]
   \[ \text{Exp: Apparent: looks like} \]
   \[ \text{dormant: hidden} \]
   \[ \text{Harbour: give shelter} \]
   \[ \text{Effect (verb): results in} \]
Q. No. 6 – 10 Carry Two Marks Each

6. A coin tossed thrice. Let X be the event that head occurs in each of the first two tosses. Let Y be the event that a tail occurs on the third toss. Let Z be the event that two tails occur in three tosses.

Based on the above information, which one of the following statements is TRUE?

(A) X and Y are not independent  
(B) Y and Z are dependent  
(C) Y and Z are independent  
(D) X and Z are independent

Answer: (B)

Exp: Let y as tail occurred in third toss and z as two tails in third toss which can be \{TTH, THT, HTT\}
y = \{TTH, TTT\}
\therefore both y and z are dependent.

7. Given below are two statements followed by two conclusions. Assuming these statements to be true, decide which one logically follows.

Statements:
I. No manager is a leader.  
II. All leaders are executives.

Conclusions:
I. No manager is an executive.  
II. No executive is a manager.

(A) Only conclusion I follows.  
(B) Only conclusion II follows.  
(C) Neither conclusion I nor II follows.  
(D) Both conclusions I and II follow.

Answer: (C)

Exp: Therefore concluding diagram can be

It can be manager that is manager can be executive also.
Some executives are also leaders that is not a manager.
8. In the given figure angle Q is a right angle, PS:QS=3:1, RT:QT=5:2 and PU:UR=1:1. If area of triangle QTS is 20 cm², then the area of triangle PQR in cm² is ________. 

![Diagram of triangle PQR with points P, Q, S, T, U, and R]

**Answer:** 280  

**Exp:** Let area of triangle PQR be ‘A’

\[
\frac{SQ}{PQ} = \frac{1}{1+3} = \frac{1}{4} \\
\frac{QT}{QR} = \frac{2}{2+5} = \frac{2}{7} \\
\therefore \text{Area of } \Delta \text{QTS} = \frac{1}{2} \times SQ \times QT
\]

\[
= \frac{1}{2} \times \left(\frac{1}{4} PQ\right) \times \left(\frac{2}{7} QR\right)
\]

\[
= \frac{1}{14} \times \text{Area of } \Delta \text{PQR}
\]

Given \(20 \text{ cm}^2 = \frac{1}{14} \times A\)

\[\therefore A = 14 \times 20 = 280 \text{ cm}^2\]

9. Select the appropriate option in place of underlined part of the sentence.

*Increased productivity necessary reflects greater efforts made by the employees.*  
(A) Increase in productivity necessary  
(B) Increase productivity is necessary  
(C) Increase in productivity necessarily  
(D) No improvement required

**Answer:** (C)

10. Right triangle PQR is to be constructed in the xy-plane so that the right angle is at P and line PR is parallel to the x-axis. The x and y coordinates of P, Q, and R are to be integers that satisfy the inequalities: \(-4 \leq x \leq 5 \text{ and } 6 \leq y \leq 16\). How many different triangles could be constructed with these properties?

(A) 110  
(B) 1,100  
(C) 9,900  
(D) 10,000

**Answer:** (C)
**Exp:**

\[ X_1 \rightarrow -4 \leq X_1 \leq 5 \]
\[ Y_1 \rightarrow 6 \leq Y_1 \leq 16 \]
\[ X_2 \rightarrow 9 \text{ chairs} (\because X_1 \neq X_2) \]
\[ Y_2 \rightarrow 10 \text{ chairs} (\because Y_1 \neq Y_2) \]
\[ \therefore \text{Total triangles} = 10 \times 11 \times 9 \times 10 = 9900 \]

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**Instrumentation Engineering**

Q. No. 1 – 25 Carry One Mark Each

1. The capacitor shown in the figure is initially charged to +10 V. The switch closes at time \( t=0 \). Then the value of \( V_c(t) \) in volts at time \( t=10 \) ms is _____ V.

![Capacitor Diagram]

**Answer:** 3.67

**Exp:** It is a source free network where capacitor voltage

\[ V_c(t) = V_0 e^{-t/\tau}; t > 0 \]
\[ = 10 e^{-100t} \]
\[ V_c(10 \times 10^{-3}) = 10 e^{-100(10 \times 10^{-3})} = 3.67V \]

2. The voltage \( (E_o) \) developed across a glass electrode for pH measurement is related to the temperature \( (T) \) by the relation

(A) \( E_o \propto \frac{1}{T} \)  
(B) \( E_o \propto \frac{1}{T^2} \)  
(C) \( E_o \propto T \)  
(D) \( E_o \propto T^2 \)

**Answer:** (C)

3. Let \( 3+4j \) be a zero of a fourth order linear –phase FIR filter. The complex number which is NOT a zero of this filter is

(A) \( 3-4j \)  
(B) \( \frac{3}{25} + \frac{4}{25}j \)  
(C) \( \frac{3}{25} - \frac{4}{25}j \)  
(D) \( \frac{1}{3} - \frac{1}{4}j \)

**Answer:** (D)

**Exp:** The property of a FIR filter is that if \( Z_o \) is a zero then the remaining zeros are
\[
\begin{align*}
\frac{1}{Z_0}, & \quad [Z_0]^*, \quad \left[\frac{1}{Z_0}\right]^* \\
\downarrow & \quad \downarrow & \quad \downarrow \\
\left[\frac{3}{25} - \frac{j4}{25}\right], & \quad \left[3 - 4j\right], \quad \left[\frac{3}{25} + \frac{j4}{25}\right]
\end{align*}
\]

So option (D) is not matching with any of this.

4. The bridge most suited for measurement of a four-terminal resistance in the range of 0.001\(\Omega\) to 0.1\(\Omega\) is
   (A) Wien’s bridge                (B) Kelvin double bridge
   (C) Maxwell’s bridge            (D) Schering bridge

Answer: (B)

5. A load resistor \(R_L\) is connected to a battery of voltage \(E\) with internal resistance \(R_i\) through a resistance \(R_s\) as shown in the figure. For fixed values of \(R_L\) and \(R_i\), the value of \(R_s\) (\(\geq 0\)) for maximum power transfer to \(R_L\) is

\[
\begin{align*}
\text{(A)} & \quad 0 \\
\text{(B)} & \quad R_L - R_i \\
\text{(C)} & \quad R_L \\
\text{(D)} & \quad R_L + R_i
\end{align*}
\]

Answer: (A)

Exp: When load is constant, we should see for what value of resistance current will be maximum and \(P_{R_L} = \text{max}\).

6. A mass-spring-damper system with force as input and displacement of the mass as output has a transfer function \(G(s) = \frac{1}{s^2 + 24s + 900}\). A force input \(F(t) = 10 \sin(70t)\) Newtons is applied at time \(t = 0s\). A beam from an optical stroboscope is focused on the mass. In steady state, the strobe frequency in hertz at which the mass appears to be stationary is
   (A) \(5/\pi\)                (B) \(15/\pi\)                (C) \(35/\pi\)                (D) \(50/\pi\)

Answer: (C)
7. A light detector circuit using an ideal photo-diode is shown in the figure. The sensitivity of the photo-diode is 0.5 μA/μW. With \( V_r = 6 \text{V} \), the output voltage \( V_o = -1.0 \text{V} \) for 10 μW of incident light. If \( V_r \) is changed to 3V, keeping all other parameters the same, the value of \( V_o \) in volts is ________ V.

![Light Detector Circuit Diagram]

**Answer:** -1

**Exp:** Here \( V_o \) is independent of the value of \( V_r \). It depends on the intensity of incident light.

8. The logic evaluated by the circuit at the output is

(A) \( XY + Y\bar{X} \)

(B) \( (\bar{X} + Y)XY \)

(C) \( \bar{X}Y + XY \)

(D) \( \bar{X}Y + X\bar{Y} + X + Y \)

**Answer:** (A)

**Exp:** Output of upper AND gate is \( XY \)

Output of lower AND gate is \( \bar{X}Y \)

Output of OR gate is \( X\bar{Y} + \bar{X}Y \).

9. The value of \( \int_{c} \frac{1}{z^2} \text{dz} \), where the contour is the unit circle traversed clockwise, is

(A) \(-2\pi i\)  

(B) 0  

(C) \(2\pi i\)  

(D) \(4\pi i\)

**Answer:** (B)

**Exp:** Given \( \int_{c} \frac{1}{z^2} \text{dz} \) where \( c \) is unit circle.

Let \( f(z) = \frac{1}{z^2} \)
f(z) is not analytic at z = 0

By Laurent series, f(z) can be expressed as

\[ f(z) = (\text{zero tens}) + \left( \frac{1}{z} + \frac{1}{z^2} \right) \]

Analytic part  Principal part

Principal part has only two terms

\[ \therefore z = 0 \text{ is pole of order 2} \]

\[ [\text{Res } f(z)]_{z=0} = \text{coefficient of } \frac{1}{z} \]

\[ = b_1 = 0 \]

\[ \therefore \text{By Cauchy's residue theorem} \]

\[ \oint \frac{1}{z} \, dz = 2\pi i \times \text{(sum of residues)} \]

\[ = 2\pi i \times 0 = 0 \]

10. Consider the ammeter-voltmeter method of determining the value of the resistance R using the circuit shown in the figure. The maximum possible errors of the voltmeter and ammeter are known to be 1% and 2% of their readings, respectively. Neglecting the effects of meter resistances, the maximum possible percentage error in the value of R determined from the measurements, is ___%.

**Answer:** 3

**Exp:**

\[ R = \frac{V_i}{I_i} = \frac{V_i}{I_i \pm 1\%} = R \pm 3\% \]

11. The highest frequency present in the signal x(t) is \( f_{\text{max}} \). The highest frequency present in the signal \( y(t) = x^2(t) \) is

(A) \( \frac{1}{2}f_{\text{max}} \)  
(B) \( f_{\text{max}} \)  
(C) \( 2f_{\text{max}} \)  
(D) \( 4f_{\text{max}} \)

**Answer:** (C)

**Exp:**

Multiplication in time domain corresponds to convolution in frequency domain

\[ x^2(t) \rightarrow x(f) \ast x(f) \]

Using limit property of convolution \( x^2(t) \) have maximum frequency \( 2f_{\text{max}} \).
12. Let A be an \( n \times n \) matrix with the rank \( r (0 < r < n) \). Then \( Ax = 0 \) has \( p \) independent solutions, where \( p \) is

(A) \( r \)  
(B) \( n \)  
(C) \( n-r \)  
(D) \( n+r \)

**Answer:** (C)

**Exp:** Given \( AX = 0 \)

\[ p(A_{n \times n}) = r \quad (0 < r < n) \]

\( p \) = no. of independent solution

i.e. nullity = \( p \)

We know that rank + nullity = \( n \)

\[ r + p = n \]

\[ \Rightarrow p = n - r \]

13. The double integral \( \int_{0}^{a} \int_{0}^{x} f(x, y) \, dy \, dx \) is equivalent to

(A) \( \int_{0}^{a} \int_{0}^{x} f(x, y) \, dx \, dy \)

(B) \( \int_{0}^{a} \int_{x}^{a} f(x, y) \, dx \, dy \)

(C) \( \int_{0}^{a} \int_{0}^{x} f(x, y) \, dy \, dx \)

(D) \( \int_{0}^{a} \int_{0}^{a} f(x, y) \, dx \, dy \)

**Answer:** (C)

**Exp:** Given double integral \( \int_{0}^{a} \int_{0}^{x} f(x, y) \, dx \, dy \)

and \( x = 0 \) to \( x = a \), \( y = 0 \) to \( y = a \)

the diagram is

\[ \int_{0}^{a} \int_{0}^{x} f(x, y) \, dy \, dx \]

By applying change of order of integration

\[ \int_{0}^{a} \int_{y}^{a} f(x, y) \, dx \, dy \]

14. In the circuit shown, the switch is momentarily closed and then opened. Assuming the logic gates to have non-zero delay, at steady state, the logic states of \( X \) and \( Y \) are
(A) X is latched, Y toggles continuously  
(B) X and Y are both latched  
(C) Y is latched, X toggles continuously  
(D) X and Y both toggle continuously  

Answer:  

Exp: The above circuit is a stable multivibrator circuit, where odd numbers of inverter are there in the loop. In such a circuit, irrespective of the position of output, it always toggles.  

→ Latching means X and Y will be fixed to same value, in this case it is not possible.

15. Consider the logic circuit with input signal TEST shown in the figure. All gates in the figure shown have identical non-zero delay. The signal TEST which was at logic LOW is switched to logic HIGH and maintained at logic HIGH. The output

(A) stays HIGH throughout  
(B) stays LOW throughout  
(C) pulses from LOW to HIGH to LOW  
(D) pulses from HIGH to LOW to HIGH

Answer:  

Exp: For analysis point of view, assume delay of each gate is 10 msec. However we can take any value.  

→ By referring the circuit the upper input to the NAND gate is direct test signal.  
→ The lower input to NAND gate is TEST but with a delay of 30 nsec.  
→ Assuming the delay of NAND gate is 0. First draw output waveform (ideal case) then shift that by 10 msec. i.e. introduce the delay.

```
Test
  0
Test
(with delay = 30nsec)
  0
Output with
delay = 0
  0
  30 n sec
Output with
NAND gate
delay = 10 n sec
  0
  10 nsec
  40 n sec
```
So we can clearly say that initial output change from high to low, then it changes from low to high and then finally at steady state output is 1.

**Note:** Saying output is high (option A) will be wrong here. We are not interested to find steady state output.

16. A p-type semiconductor strain gauge has a nominal resistance of 1000 $\Omega$ and a gauge factor of +200 at 25° C. The resistance of the strain gauge in ohms when subjected to a strain of $+10^{-4}$ m/m at the same temperature is _________ $\Omega$.

**Answer:** 1020

**Exp:**

\[ GF = \frac{\Delta R / R}{\epsilon} \Rightarrow \Delta R = 200 \times 10^{-4} \times 1000 \]

\[ = 20 \Omega \]

P-type semiconductor have a positive gauge factor

i.e., Resistance $\uparrow$ with $\uparrow$ in strain

$\therefore R_{\text{strained}} = 1000 + 20 = 1020 \Omega$

17. A system with transfer function $G(s) = \frac{1}{s^2 + 1}$ has zero initial conditions. The percentage overshoot in its step response is _________ %.

**Answer:** 100

**Exp:**

Comparing the denominator, $\zeta = 0$

$\% \text{ Overshoot} = e^{\frac{\pi}{\sqrt{\xi}}} \times 100 = 100$

18. Liquid flow rate is measured using

(A) a Pirani gauge   
(B) a pyrometer   
(C) an orifice plate   
(D) a Bourdon tube

**Answer:** (C)

19. The magnitude of the directional derivative of the function $f(x, y) = x^2 + 3y^2$ in a direction normal to the circle $x^2 + y^2 = 2$, at the point (1, 1), is

(A) $4\sqrt{2}$   
(B) $5\sqrt{2}$   
(C) $7\sqrt{2}$   
(D) $9\sqrt{2}$

**Answer:** (A)

**Exp:**

Let $f(x, y) = x^2 + 3y^2$

Let $\phi = x^2 + y^2 - 2$ and P is (1, 1)

Normal to the surface $\phi = \nabla \phi$

\[ = \frac{\partial \phi}{\partial x} + j \frac{\partial \phi}{\partial y} \]

\[ = 2x + 2y \]

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\[ \nabla \phi \big|_{a_{(1,1)}} = 2i + 2j \]

Let \( \vec{a} = \nabla \phi = 2i + 2j \)

We need to calculate magnitude of directional derivatives of \( f \) along \( \vec{a} \) at (1,1)

Magnitude of directional deviations = \( \nabla f \cdot \vec{a} \)

\[ \nabla f = \frac{\partial f}{\partial x} + j \frac{\partial f}{\partial y} = 2x i + 6y j \]

\[ \nabla f \big|_{(1,1)} = 2i + 6j \]

\[ |a| = \sqrt{4 + 4} = 2\sqrt{2} \]

\[ \hat{a} = \frac{2i + 2j}{2\sqrt{2}} = \frac{i + j}{\sqrt{2}} \]

Magnitude of direction derivative \( = (2i + 6j) \left( \frac{i + j}{\sqrt{2}} \right) \)

\[ = \frac{2 + 6}{\sqrt{2}} = \frac{8}{\sqrt{2}} = 4\sqrt{2} \]

20. A power line is coupled capacitively through various parasitic capacitances to a shielded signal line as shown in the figure. The conductive shield is grounded solidly at one end. Assume that the length of the signal wire extending beyond the shield, and the shield resistance are negligible. The magnitude of the noise voltage coupled to the signal line is

(A) directly proportional to \( C_{1G} \)

(B) inversely proportional to the power line frequency

(C) inversely proportional to \( C_{15} \)

(D) Zero

Answer: (D)
21. The torque transmitted by a cylindrical shaft is to be measured by using two strain gauges. The angles for mounting the strain gauges relative to the axis of the shaft for maximum sensitivity are

(A) $\pm 45^\circ$  
(B) $\pm 60^\circ$  
(C) $\pm 90^\circ$  
(D) $\pm 180^\circ$

Answer:  (A)

Exp:

![Diagram showing the angles for mounting the strain gauges relative to the axis of the shaft.]

22. The output voltage of the ideal transformer with the polarities and dots shown in the figure is given by

(A) $N V_1 \sin \omega t$  
(B) $-N V_1 \sin \omega t$  
(C) $\frac{1}{N} V_1 \sin \omega t$  
(D) $-\frac{1}{N} V_1 \sin \omega t$

Answer:  (B)

Exp: First mark the mutual voltage polarity using dot convention (when a reference current enters at dot of one coil, it generates positive polarity on dot terminal of other coil)

Using Transformer ratio

\[ \frac{V_2}{V_1} = \frac{N_2}{N_1} \]

\[ V_2 = V_1 \left( \frac{N}{1} \right) = N V_1 \sin \omega t \]

\[ V_o = -V_2 = -N V_1 \sin \omega t \]

23. The filter whose transfer function is of the form \( G(s) = \frac{s^2 - bs + c}{s^2 + bs + c} \) is

(A) a high-pass filter  
(B) a low-pass filter  
(C) an all-pass filter  
(D) a band-reject filter

Answer:  (C)
24. An apparatus to capture ECG signals has a filter followed by a data acquisition system. The filter best suited for this application is
(A) low pass with cutoff frequency 200 Hz
(B) high pass with cutoff frequency 200 Hz
(C) band pass with lower and upper cutoff frequencies 100 Hz and 200 Hz for its pass band
(D) band reject with lower and upper cutoff frequencies 1 Hz and 200 Hz for its stop band

Answer: (A)

Exp: More appropriate will be a Band pass filter from 1Hz to 200Hz

25. The figure shows a half-wave rectifier circuit with input voltage \( V(t) = 10\sin(100\pi t) \) volts. Assuming ideal diode characteristics with zero forward voltage drop and zero reverse current, the average power consumed in watts by the load resistance \( R_L \) is ____. W.

Answer: 0.25

Exp: The voltage waveform at \( R = 100\Omega \) is as shown below.

\[
P_{avg} = \frac{V_{rms}^2}{R} = \frac{(10/2)^2}{100} = 0.25 \text{ Watt}
\]

Q. No. 26 – 55 Carry Two Marks Each

26. Consider a low-pass filter module with a pass-band ripple of \( \delta \) in the gain magnitude. If M such identical modules are cascaded, ignoring the loading effects, the pass-band ripple of the cascade is
(A) \( 1-(1-\delta)^M \)
(B) \( \delta^M \)
(C) \( (1-\delta)^M \)
(D) \( (1-\delta)^M \)

Answer: A

27. The probability density function of a random variable \( X \) is \( p_X(x) = e^{-x} \) \( x \geq 0 \) and 0 otherwise. The expected value of the function \( g(x) = e^{3x/4} \) is ________.

Answer: 4

Exp: Probability density function of \( X \) is
given as \( P(x) = e^{-x}; \ x \geq 0 \)
\( = 0; \) otherwise
and \( g(x) = e^{\frac{3x}{2}} \)

\[ E(g(x)) = \int_{0}^{\infty} f(x)g(x)dx \]
\[ = \int_{0}^{\infty} e^{-x}e^{\frac{3x}{2}}dx \]
\[ = \int_{0}^{\infty} e^{\frac{x}{2}}dx \]
\[ = \int_{0}^{\infty} e^{\frac{x}{2}}dx \]
\[ = \left[ \frac{e^{x/4}}{1/4} \right]_{0}^{\infty} \]
\[ = -4[e^{-\infty} - 1] \]
\[ = 4(0 - 1) = 4 \]

28. A system is represented in state-space as \( \dot{X} = AX + Bu \), where \( A = \begin{bmatrix} 1 & 2 \\ \alpha & 6 \end{bmatrix} \) and \( B = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \).

The value of \( \alpha \) for which the system is not controllable is _________.

Answer: -3

Exp: For a system to be uncontrollable, its controllability determinant should be equal to zero.

\[ Q_c = |B \ AB| = 0 \]

\[ AB = \begin{bmatrix} 1 & 2 \\ \alpha & 6 \end{bmatrix} \begin{bmatrix} 1 \\ \alpha + 1 \end{bmatrix} = \begin{bmatrix} 3 \\ \alpha + 6 \end{bmatrix} \]

\[ Q_c = |B \ AB| \rightarrow \begin{bmatrix} 3 \\ \alpha + 6 \end{bmatrix} = 0 \Rightarrow \alpha + 6 - 3 = 0 \Rightarrow \alpha = -3 \]

29. The Seebeck coefficients, in \( \mu V/\degree C \), for copper, constantan and iron, with respect to platinum, are 1.9, -38.3 and 13.3 respectively. The magnitude of the thermo emf \( E \) developed in the circuit shown in the figure, in millivolts is __________mV.

The magnitude of the thermo emf \( E \) developed in the circuit shown in the figure, in millivolts is __________mV.
Answer: 1.236

Exp:

\( V_{\text{Cu-Constantan}} = 1.9 - (-38.3) \) \\
\( = 40.2 \mu V / \degree C \)

\( V_{\text{Cu-Constantan}} = 40.2 \times 50 = 2010 \mu V = 2.01 mV \)

\( V_{\text{Fe-Constantan}} = 13.3 - (-38.3) = 51.6 \mu V / \degree C \)

\( V_{\text{Fe-Constantan}} = 51.6 \times 15 = 0.774 mV \)

\( V_{\text{out}} = 2.01 - 0.774 = 1.236 mV \)

30. The current in amperes through the resistor R in the circuit shown in the figure is __________.

\[ \begin{align*}
\text{Answer:} & \quad 1 \\
\text{Exp:} & \quad \text{Using supermesh analysis} \\
& \quad \text{Mesh 1, 3 form super mesh} \\
& \quad 2I_1 - I_2 + 2I_1 - I_2 = 0 \\
& \quad \Rightarrow 2I_1 - 2I_2 + 2I_1 = 0 \quad \cdots (1) \\
& \quad \text{Writing KCL at Q} \\
& \quad I_1 - I_3 = -1 \quad \cdots (2) \\
& \quad \text{Writing KCL on mesh 2} \\
& \quad 2I_2 - I_1 - I_3 = 1 \quad \cdots (3) \\
& \quad \text{Solution Equation 1, 2, 3} \\
& \quad I_1 = 0, I_2 = I_3 = 1 A \\
& \quad \text{Current through R is} \quad 1 A \\
& \quad \text{Note:} \quad \text{Strictly saying it is an ambiguous question as the direction of current is not mentioned, so it could be} -1 A \text{ as well.} \\
\end{align*} \]
31. In the circuit shown, the voltage source \( V(t) = 15 + 0.1 \sin(100t) \) volts. The PMOS transistor is biased such that it is in saturation with its gate-source capacitance being 4nF and its transconductance at the operating part being 1 mA/V. Other parasitic impedances of the MOSFET may be ignored. An external capacitor of capacitance 2 nF is connected across the PMOS transistor as shown. The input impedance in mega ohm as seen by the voltage source is \( \boxed{1} \) MΩ.

![Circuit Diagram]

**Answer:** 1

32. The probability that a thermistor randomly picked up from a production unit is defective is 0.1. The probability that out of 10 thermistors randomly picked up, 3 are defective is
(A) 0.001
(B) 0.057
(C) 0.0107
(D) 0.3

**Answer:** (B)

**Exp:**
\[
p = \text{Probability of a thermistor is defective} = 0.1
\]
\[
q = 1 - p = 1 - 0.1 = 0.9
\]
\[
n = 10
\]

Let \( X \) be random variable which is no. of defective pieces.

Required probability = \( P(X = 3) \)
\[
= ^{10}C_3 p^3 q^7
\]
\[
= ^{10}C_3 (0.1)^3 (0.9)^7
\]
\[
= 0.057
\]

33. A liquid level measurement system employing a radio-isotopic is mounted on a tank as shown in the figure. The absorption coefficient of water for the radiation is 7.7 m\(^{-1}\). If the height of water in the tank is reduced from 100 mm to 90 mm, the percentage change in the radiation intensity received by the detector, neglecting absorption of the radiation by air, is \( \boxed{____} \)%.
Answer: 8

Exp: General formula for the amount of intensity received, \( I = I_0 e^{-\mu \rho x} \)

- \( I_0 \rightarrow \) Intensity when absorbing material is nil or empty
- \( \mu \rightarrow \) absorption coefficient
- \( \rho \rightarrow \) density of the absorption material
- \( x \rightarrow \) thickness (or) height of the absorbing material

By considering the given parameters and neglecting others

Intensity received,

(a) When \( x \rightarrow \) height = 100 mm = 0.1 m
\[
I_1 = e^{-7.7 \times 0.1} = e^{-0.77}
\]

(b) When \( x = 90 \) mm = 0.9 m
\[
I_2 = e^{-7.7 \times 0.09} = e^{-0.693}
\]

So, \( \% \) change = \( \frac{\text{change}}{\text{Initial}} \times 100 = \frac{I_2 - I_1}{I_1} \times 100 \geq 8\%

34. A beam of monochromatic light passes through two glass slabs of the same geometrical thickness at normal incidence. The refractive index of the first slab is 1.5 and that of the second, 2.0. The ratio of the time of passage of the beam through the first to the second slab is _________.

Answer: 0.75

Exp: We know, \( \text{time} = \frac{\text{distance}}{\text{velocity}} = \frac{d}{v} \)
The ratio of the time for 2 slabs \( \frac{t_1}{t_2} \)

\[
\frac{d_1}{v_1} = \frac{d_1 \times v_2}{v_1} \quad \text{because of same geometrical thickness}
\]

\[
= \frac{v_2}{v_1} = \frac{\mu_2}{\mu_1}
\]

\[
= \frac{1.5}{2.0} = 0.75
\]

NOTE: If a light of velocity “v” is passing through a matter of R.I = 0.7 (say)

Then the final output velocity will be \( \frac{v}{0.7} \)

35. In the circuit shown in the figure, it is found that \( V_{BE} = 0.7 \) V amnd \( V_E = 0V \). If \( \beta_{dc} = 99 \) for the transistor, then the value of \( R_B \) in kilo ohms is __________ k\( \Omega \).

[Diagram of a circuit with labels R_B, V_E = 0V, I_E, 10k\( \Omega \), -10V, +10V]

Answer: 93
Exp:

\[ I_B = \frac{10 - 0.7}{R_B} \]  

\[ I_E = (\beta + 1)I_B = \frac{0 - (-10V)}{1k} = 10mA \]

\[ \therefore I_B = \frac{10}{100} mA \quad (\therefore \beta = 99) \]

\[ = 100\mu A \]

Put the volume of \( I_B \) in equation (1), we get

\[ R_B = \frac{9.3}{I_B} = \frac{9.3}{100} \times 10^6 = 93k\Omega \]

36. The output frequency of an LC tank oscillator employing a capacitive sensor acting as the capacitor of the tank is 100 kHz. If the sensor capacitance increases by 10%, the output frequency in kilohertz becomes ________ kHz.

Answer: 95.35

Exp:

\[ f_1 = \frac{1}{2\pi\sqrt{LC}} \]

Now \( C' = C + 0.1C = 1.1C \)

\[ f_2 = \frac{1}{2\pi\sqrt{1.1LC}} = \frac{f}{\sqrt{1.1}} \]

\[ \therefore f_2 = \frac{100kHz}{\sqrt{1.1}} = 95.346 kHz \]

37. The resolving power of a spectrometer consisting of a collimator, a grating and a telescope can be increased by

(A) increasing the angular magnification of the telescope

(B) increasing the period of the grating

(C) decreasing the period of the grating

(D) decreasing the slit-width of the collimator

Answer: (C)

Exp: Resolving power of a grating plate can be increased by increasing the number of slits. i.e., by decreasing the grating period.
38. A transfer function \( G(s) \) with the degree of its numerator polynomial zero and the degree of its denominator polynomial two has a Nyquist plot shown in the figure. The transfer function represents

\[
\text{Im}(G(j\omega))
\]

\[
\text{Re}(G(j\omega))
\]

(A) a stable, type-0 system

(B) a stable, type-1 system

(C) an unstable, type-0 system

(D) an unstable, type-1 system

**Answer:** (D)

**Exp:** Consider the Transfer function to be

\[ G(s) = \frac{1}{s(s-1)} \]

Magnitude \( M = \frac{1}{\sqrt{\omega^2 + 1}} \), \( \phi = -270^\circ + \tan^{-1}(\omega) \)

\( \omega = 0 \quad M = \infty, \quad \phi = -270^\circ \)

\( \omega = \infty \quad M = 0, \quad \phi = -270^\circ + 90^\circ = -180^\circ \)

\[ \therefore \text{our assumption of } G(s) \text{ is true} \]

The system is type-1 and is unstable.

39. For the circuit shown in the figure, the rising edge triggered D-flip flop with asynchronous reset has a clock frequency of 1 Hz. The NMOS transistor has an ON resistance of 1000\( \Omega \) and an OFF resistance of infinity. The nature of the output waveform is
Answer:  (A)

Exp:  Before solving the problem, consider the following fact.

→ clock is +ve edge triggered and input to D flip-flop is constant i.e logic 1.

→ Reset pin of Flip flip is active high and it can function at any moment of time independent to clock. Also note that reset will be triggered when $V^+$ terminal voltage just crosses 0.1 V.

→ N-MOS behave as open switch (off state, $R = \infty$) when $\bar{Q} = 0$ and it will behave as a resistance of 1000$\Omega$ when $\bar{Q} = 1$ (as ON resistance is specified).

→ Now for analysis, assume capacitor is initially uncharged and a +ve edge is triggered to D-flipflop, as its input is high. Then $Q=1$ and $\bar{Q}=0$.

* When $\bar{Q} = 0$, N-MOS is off, assuming the input resistance of comparator infinite. Then the source current can flow only through the capacitor. Now we have to see how much time the capacitor will take to reach 0.1 V. So that $V^+ > V^-$ and $R$ will change from 0 to 1 and as the reset terminal will be triggered the state of flip-flop changes i.e. $(Q=0, \bar{Q}=1)$ and N-MOS gets ON. As the integration of step is ramp if we integrate the constant current voltage will be ramp.

→ For this use the capacitor voltage equation.

$$V_c(t) = \frac{1}{C} \int_{t_0}^{t} i_\tau d\tau$$

$$0.1V = \frac{1}{2 \times 10^{-6}} \int_{t_0}^{t} 1 \times 10^{-6} d\tau$$

$$0.1 = \frac{t}{2} \Rightarrow t = 0.2\text{ sec} = 200\text{ milli sec}$$

\[\text{Diagram of voltage and current}\]
So at $t = 0.2\text{sec}$, N-MOS is ON. Now the circuit is equivalent to a current source connected to a parallel combination of RC with non-zero initial voltage of capacitor (0.1V).

Now the capacitor will discharge from its initial voltage with time constant $\tau = RC = 2\text{ms}$ and at $10\tau$ it will react steady state i.e., at 20 m.sec. The nature of discharge is exponential decay.

So we can conclude from the above analysis that capacitor takes 200 msec to charge up to 0.1V and 20 msec to discharge to the steady state value.

So if the +ve edge of clock appeared at $t = 0$ then at 200 msec it completed its charging and at 220 msec it completed its discharging. This continues in each clock cycles.

The following waveform gives further clarity as $f_{\text{CLK}} = 1\text{Hz}, T_{\text{CLK}} = 1000\text{ms}$

40. Consider the circuits shown in the figure. The magnitude of the ratio of the currents, i.e., $|I_1/I_2|$, is ____________.

Answer: 1

Exp: By reciprocity theorem, $I_1 = I_2$
41. The linear I-V characteristics of 2-terminal non-ideal dc sources X and Y are shown in the figure. If the sources are connected to a 1Ω resistor as shown, the current through the resistor in amperes is ________A.

Answer: 1.75

Exp: The transfer characteristic curve and the circuit of a non ideal source is given below.

Comparing the curve of X and Y we can conclude that

\[
X: \quad V_x = 4V, \quad R_x = 2\Omega \\
Y: \quad V_y = 3V, \quad R_y = 1\Omega
\]

When we connect these the circuit becomes,

\[
I_{1A} = \frac{\frac{4}{2} + \frac{3}{1}}{1 + 1} = 1.75\ \text{A}
\]

42. The figure shows a spot of light of uniform intensity 50 W/m² and size 10mm×10 mm incident at the exact centre of a photo-detector, comprising two identical photo-diodes D₁ and D₂. Each diode has a sensitivity of 0.4 A/W and is operated in the photoconductive mode. If the spot of light is displaced upwards by 100μm, the resulting difference between the photocurrents generated by D₁ and D₂ in micro amperes, is ________ μA.
Answer: 40

Exp: Total illuminated Area = 10×10×10^-6 m^2 = 10^-4 m^2
Light power = 50×10^-4 W = 5mW
Output from both diodes = 0.4×5mA = 2mA
So output from each diode = 1mA
In second case,
Area of one diode (D_1) = 10×5.1×10^-6
= 0.51×10^-4 m^2
Light output (D_1) = 0.51×10^-4×50W
= 2.55 mW
Current in D_1 = 2.55×0.4 = 1.02 mA
Current in D_2 = Total current – I_{D1}
= 2-1.02 = 0.98 mA
∴ I_{D1} – I_{D2} = (1.02 – 0.98) mA = 0.04 mA = 40μA
43. An ADC is interfaced with a microprocessor as shown in the figure. All signals have been indicated with typical notations. Acquisition of one new sample of the analog input signal by the microprocessor involves

(A) One READ cycle only
(B) One WRITE cycle only
(C) One WRITE cycle followed by one READ cycle
(D) One READ cycle followed by one WRITE cycle

Answer: (C)

44. In the figure shown, \( R_T \) represents a resistance temperature device (RTD), whose characteristic is given by \( R_T = R_0 (1+\alpha T) \), where \( R_0 = 100\,\Omega \), \( \alpha = 0.0039^\circ\text{C}^{-1} \) and \( T \) denotes the temperature in \( ^\circ\text{C} \). Assuming the op-amp to be ideal, the value of \( V_o \) in volts when \( T = 100^\circ\text{C} \), is _______ V.

\[
V_i = +1V
\]

\[
R_T
\]

\[
100\,\Omega
\]

\[
V_o
\]

Answer: 1.39

Exp: Using the above information
\[ R_T = R_x \left[ 1 + \alpha T \right] = 100 \left[ 1 + 0.0039 \times 100 \right] = 139 \Omega \]

→ Since it is a non inverting amplifier its output voltage w.r.t ground is
\[ \left( 1 + \frac{R_T}{R_x} \right) V = \left( 1 + \frac{139}{100} \right) = 2.39V \]

→ But here \( V_o \) is not w.r.t ground, it is across \( R_T \)
So \( V_o = 2.39 - 1 = 1.39V \)

45. The signal \( x[n] = \sin(\pi n/6)/(\pi n) \) is processed through a linear filter with the impulse response \( h[n] = \sin(\omega_c n)/(\pi n) \) where \( \omega_c > \pi/6 \). The output of the filter is

(A) \( \sin(2\omega_c n)/(\pi n) \)  
(B) \( \sin(\pi n/3)/(\pi n) \)  
(C) \( [\sin(\pi n/6)/(\pi n)]^2 \)  
(D) \( [\sin(\pi n/6)/(\pi n)] \)

Answer: (D)

Exp:
\[
\left[ \frac{\sin \alpha n}{\pi n} \right] \times \left[ \frac{\sin \beta n}{\pi n} \right] = \left[ \frac{\sin \gamma n}{\pi n} \right]
\]
where \( \gamma = \min(\alpha, \beta) \)

Output \( g(n) = x(n) \ast h(n) \)

In frequency domain they will be multiplied.

46. If the deflection of the galvanometer in the bridge circuit shown in the figure is zero, then the value of \( R_x \) in ohms is \( \underline{33.33} \Omega \).

Exp: When the current through galvanometer is 0 then its 2 end voltage are same or we can say the open circuit voltage across galvanometer is 0.
By voltage division, \( V_s = 0.5V \)

\[
V_b = \left( \frac{2}{100 + R_x} \right) R_x
\]

\( V_a = V_b \)

\[
\frac{1}{2} = \frac{2R_x}{100 + R_x} \Rightarrow 2R_x = 100 + R_x \Rightarrow R_x = \frac{100}{3} = 33.33\Omega
\]

47. In the circuit shown in the figure, both the NMOS transistors are identical with their threshold voltages being 5V. Ignoring channel length modulation, the output voltage \( V_{out} \) in volt is \( \underline{20} \) V.

Answer: \( 20 \)

Exp: \( V_{GS1} = V_{GS2} \) (Both are identical transistors)

\[
30 - V_{out} = 10 - 0
\]

\[
V_{out} = 20V
\]
48. In the potentiometer circuit shown in the figure, the expression for $V_x$ is

![Potentiometer Circuit](image)

(A) $(1 - 2\alpha)V$  
(B) $(1 - \alpha)V$  
(C) $(\alpha - 1)V$  
(D) $\alpha V$

**Answer:** (A)

**Exp:** Writing KVL as per the current direction

$$V_x + (\alpha R)\frac{2V}{R} - V = 0$$

49. An opamp has ideal characteristics except that its open loop gain is given by the expression $A_v(s) = \frac{10^4}{1 + 10^{-3}s}$. This op-amp is used in the circuit shown in the figure. The 3-dB bandwidth of the circuit, in rad/s, is

![Opamp Circuit](image)

(A) $10^2$  
(B) $10^3$  
(C) $10^4$  
(D) $10^6$

**Answer:** (D)

**Exp:** Closed loop transfer function $A_{CL}$

$$A_{CL} = \frac{A_{OL}}{1 + A_{OL}\beta}$$

Here $A_{OL} = \frac{10^4}{1 + 10^{-3}s}$, $\beta = \frac{R_i}{R_i + R_f} = \frac{1}{9 + 1} = 0.1$
A signal is band-limited to 0 to 12 kHz. The signal spectrum is corrupted by additive noise which is band-limited to 10 to 12 kHz. Theoretically, the minimum rate in kilohertz at which the noisy signal must be sampled so that the \textbf{UNCORRUPTED PART} of the signal spectrum can be recovered, is \_\_\_\_\_\_\_kHz.

\textbf{Answer:} 22

\textbf{Exp:}

The signal is corrupted from 10 to 12 kHz. So the ideal sampling,

\[ X_s(f) = \sum_{n=-\infty} X(f - nf_s) \]

\[ \rightarrow \text{If } F_s = 20 \text{ kHz then the spectrum will be} \]

So the overlapping is starting from 8 kHz onwards, but we want it should be from 10 kHz onward because the uncorrupted is up to 10 kHz.

\[ \rightarrow \text{So the sampling frequency should increase by 2kHz.} \]

So \( f_s = 20 + 2 = 22 \text{kHz} \). the spectrum.
51. The circuit shown in the figure is in series resonance of frequency $f_c$ Hz. The value of $V_c$ in volts is ______________V.

\[ V_c = QV \]

\[ Q = \frac{1}{R} \sqrt{\frac{L}{C}} = \frac{1}{10} \sqrt{\frac{0.1}{0.1 \times 10^{-6}}} = 100 \]

\[ V_c = QV = (100)(1) = 100V \]

52. The fundamental period of the signal $x(t) = 2\cos\left(\frac{2\pi t}{3}\right) + \cos(\pi t)$, in seconds, is ____________s.

\[ \omega_0 = \frac{H.C.F(2\pi, \pi)}{L.C.M(3, 1)} = \frac{\pi}{3} \]

\[ \frac{2\pi}{T} = \frac{\pi}{3} \Rightarrow T = 6 \text{ sec} \]

53. The open loop transfer function of a system is $G(s) = \frac{s^2 + 6s + 10}{s^2 + 2s + 2}$. The angles of arrival of its root loci are

\[ (A) \pm \frac{\pi}{4} \quad (B) \pm \frac{\pi}{3} \quad (C) \pm \frac{\pi}{2} \quad (D) \pm \frac{5\pi}{6} \]

Answer: (A)
Exp: Angle of arrival is calculated on a complex zero and it is given by,

\[ \phi_a = 180 - |\text{GH} \] (at a + ve imaginary zero)

\[ G(s) = \frac{(s + 3 + i)(s + 3 - i)}{(s + 1 + i)(s + 1 - i)} \]

\[ G(-3 + i) = \frac{[-3 + i + 3 + i][-3 + i + 3 - i]}{[-3 - i + 1 + i][-3 - i + 1 - i]} = \frac{[2i]}{[-2 + 2i][-2]} \]

\[ G(-3 + i) = 90° - \left[180° - \tan^{-1}\frac{2}{2}\right] - [180°] \]

\[ = 90° - 180° + 45° - 180° = 135° \]

\[ \phi_a = 180° - 135° = 45° = \frac{\pi}{4} \]

Other angle will be same with opposite sign \( \pm \frac{\pi}{4} \)

54. The \( z \)-transform of \( x[n] = a^n \), \( 0 < |a| < 1 \), is \( X(z) \). The region of convergence of \( X(z) \) is

(A) \( |z| < |\alpha| < \frac{1}{|\alpha|} \)

(B) \( |z| > a \)

(C) \( |z| > \frac{1}{|z|} \)

(D) \( |z| < \min \left[ \frac{1}{|z|}, \frac{1}{|\alpha|} \right] \)

Answer: (A)

Exp: \( x(n) = a^n = a^n u(n) + \left( \frac{1}{a} \right)^n u(-n-1) \)

\[ |z| > \alpha \; \text{:} \; |z| < \frac{1}{\alpha} \]
55. The number of clock cycles for the duration of an input pulse is counted using a cascade of N decade counters (DC 1 to DC N) as shown in the figure. If the clock frequency in mega hertz is \( f \), the resolution and range of measurement of input width, both in \( \mu s \), are respectively

\[
\begin{align*}
\text{(A)} & \quad \frac{1}{f} \text{ and } \frac{2^{N}-1}{f} \\
\text{(B)} & \quad \frac{1}{f} \text{ and } \frac{10^{N}-1}{f} \\
\text{(C)} & \quad \frac{10^{N}}{f} \text{ and } \frac{(10^{N} - 1)}{f} \\
\text{(D)} & \quad \frac{2^{N}}{f} \text{ and } \frac{2^{N} - 1}{f}
\end{align*}
\]

**Answer:** (B)

**Exp:**

The Resolution (R) is the smallest change that is detectable.

\[
\therefore R = \frac{1}{f \text{ (MHz)}} = T_{CLK}
\]

Range of measurement of input width = T
\[ T = (10^N - 1)T_{CLK} \]
\[ T = \frac{10^N - 1}{f} \]