General Aptitude

Q. 1 – Q. 5 carry one mark each

1. Out of the following four sentences, select the most suitable sentence with respect to grammar and usage.
   (A) I will not leave the place until the minister does not meet me.
   (B) I will not leave the place until the minister doesn’t meet me.
   (C) I will not leave the place until the minister meet me.
   (D) I will not leave the place until the minister meets me.
   Key: (D)

2. A rewording of something written or spoken is a ___________.
   (A) paraphrase            (B) paradox                 (C) paradigm               (D) paraffin
   Key: (A)

3. Archimedes said, “Give me a lever long enough and a fulcrum on which to place it, and I will move the world.”
   The sentence above is an example of a ________ statement.
   (A) figurative  (B) collateral (C) literal (D) figurine
   Key: (A)

4. If ‘relftaga’ means carefree, ‘otaga’ means careful and ‘fertaga’ means careless, which of the following could mean ‘aftercare’?
   (A) zentaga                 (B) tagafer                   (C) tagazen                  (D) relffer
   Key: (C)

5. A cube is built using 64 cubic blocks of side one unit. After it is built, one cubic block is removed from every corner of the cube. The resulting surface area of the body (in square units) after the removal is.
   (A) 56                          (B) 64                          (C) 72                          (D) 96
   Key: (D)

Exp: Four blocks are needed for each direction(totally 3 directions) to build a bigger cube containing 64 blocks. So area of one side of the bigger cube= $4 \times 4 = 16$ units

There are 6 faces so total area= $6 \times 16 = 96$ units

When cubes at the corners are removed they introduce new surfaces equal to exposes surfaces so the area of the bigger cube does not change from 96
Q. No. 6 – 10 Carry Two Marks Each

6. A shaving set company sells 4 different types of razors, Elegance, Smooth, Soft and Executive.

Elegance sells at Rs. 48, Smooth at Rs. 63, Soft at Rs. 78 and Executive at Rs. 173 per piece. The table below shows the numbers of each razor sold in each quarter of a year.

<table>
<thead>
<tr>
<th>Quarter/Product</th>
<th>Elegance</th>
<th>Smooth</th>
<th>Soft</th>
<th>Executive</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q1</td>
<td>27300</td>
<td>20009</td>
<td>17602</td>
<td>9999</td>
</tr>
<tr>
<td>Q2</td>
<td>25222</td>
<td>19392</td>
<td>18445</td>
<td>8942</td>
</tr>
<tr>
<td>Q3</td>
<td>28976</td>
<td>22429</td>
<td>19544</td>
<td>10234</td>
</tr>
<tr>
<td>Q4</td>
<td>21012</td>
<td>18229</td>
<td>16595</td>
<td>10109</td>
</tr>
</tbody>
</table>

Which product contributes the greatest fraction to the revenue of the company in that year?

(A) Elegance  (B) Executive  (C) Smooth  (D) Soft

Key: (B)

Exp: Total income from Elegance=48(27300+25222+28976+21012) = 4920480
Total income from Smooth=63(20009+19392+22429+18229) = 5043717
Total income from Soft=78(17602+18445+19544+16595) =5630508
Total income from Executive=173(9999+8942+10234+10109) =6796132

7. Indian currency notes show the denomination indicated in at least seventeen languages. If this is not an indication of the nation’s diversity, nothing else is.

Which of the following can be logically inferred from the above sentences?

(A) India is a country of exactly seventeen languages.
(B) Linguistic pluralism is the only indicator of a nation’s diversity.
(C) Indian currency notes have sufficient space for all the Indian languages.
(D) Linguistic pluralism is strong evidence of India’s diversity.

Key: (D)

8. Consider the following statements relating to the level of poker play of four players P, Q, R and S.

I. P always beats Q
II. R always beats S

Consider the following statements relating to the level of poker play of four players P, Q, R and S.
III. S loses to P only sometimes
IV. R always loses to Q

Which of the following can be logically inferred from the above statements?

(i) P is likely to beat all the three other players
(ii) S is the absolute worst player in the set

(A) (i) only                     (B) (ii) only                     (C) (i) and (ii)                     (D) neither (i) nor (ii)

Key: (D)

9. If \( f(x) = 2x^7 + 3x - 5 \), which of the following is a factor of \( f(x) \)?

(A) \((x^3 + 8)\)                     (B) \((x - 1)\)                     (C) \((2x - 5)\)                     (D) \((x + 1)\)

Key: (B)

Exp: from the option \(b\) substitute \(x = 1\) in

\[
2x^7 + 3x - 5 = 0 \\
2(1^7) + 3(1) - 5 = 0 \\
5 - 5 = 0 \\
\text{So } (x-1) \text{ is a factor of } f(x)
\]

10. In a process, the number of cycles to failure decreases exponentially with an increase in load. At a load of 80 units, it takes 100 cycles for failure. When the load is halved, it takes 10000 cycles for failure. The load for which the failure will happen in 5000 cycles is

(A) 40.00                    (B) 46.02                    (C) 60.01                    (D) 92.02

Key: (B)

Exp: From the data given we assume

\[
\text{load} = \frac{\text{exponent}}{\log(\text{cycles})} \\
80 = \frac{x}{\log(10000)} \Rightarrow x = 160 \\
40 = \frac{x}{\log(10000)} \Rightarrow x = 160 \\
\text{load} = \frac{160}{\log 5000} = 43.25
\]
1. Let p, q, r, s represent the following propositions.

p: \( x \in \{8, 9, 10, 11, 12\} \)

q: \( x \) is a composite number

r: \( x \) is a perfect square

s: \( x \) is a prime number

The integer \( x \geq 2 \) which satisfies \( \neg(( p \Rightarrow q) \land (\neg r \lor \neg s)) \) is.

Key: (11)

Exp:

\[ \neg(( p \Rightarrow q) \land (\neg r \lor \neg s)) = \neg (p \Rightarrow q) \lor \neg (\neg r \land \neg s) \]

\[ = \neg (p \Rightarrow q) \lor (r \land s) \]

\[ = (p \land \neg q) \lor (r \land s) \]

\[ = \neg (p \lor q) \land (r \land s) \]

\[ = (p \lor q) \lor (r \land s) \]

For \( x=11 \) only the above compound proposition is true.

2. Let \( a_n \) be the number of \( n \)-bit strings that do NOT contain two consecutive 1s. Which one of the following is the recurrence relation for \( a_n \)?

(A) \( a_n = a_{n-1} + 2a_{n-2} \)

(B) \( a_n = a_{n-1} + 2a_{n-2} \)

(C) \( a_n = a_{n-1} + 2a_{n-2} \)

(D) \( a_n = a_{n-1} + 2a_{n-2} \)

Key: (B)

Exp: Case I First bit is ‘0’

\[
\begin{array}{c}
0 \\
\hline
a_{n-1}
\end{array}
\]

it must be zero

Case II First bit is ‘1’

\[
\begin{array}{c}
1 \\
\hline
a_{n-2}
\end{array}
\]

\[ \therefore \ a_n = a_{n-1} + a_{n-2} \]
3. \( \lim_{x \to 4} \frac{\sin(x - 4)}{x - 4} = \underline{\quad}. \)

Key: (1)

Exp:
\[
\lim_{x \to 4} \frac{\sin(x - 4)}{x - 4} = \lim_{x \to 4} \frac{\sin(x - 4)}{x - 4} = \lim_{y \to 0} \frac{\sin y}{y} \quad \text{(By taking } y = x - 4) \\
= 1
\]

4. A probability density function on the interval \([a, 1]\) is given by \(1/x^2\) and outside this interval the value of the function is zero. The value of \(a\) is____.

Key: (0.5)

Exp: Given \(f(x) = \frac{1}{x^2}\) \(x \in [a, 1]\)
\(= 0\) otherwise

We know that \(\int_a^1 f(x)dx = 1\)

\[
\Rightarrow \int_a^1 \frac{1}{x^2}dx = 1 \Rightarrow \left(\frac{-1}{x}\right)_a = 1 \\
\Rightarrow \frac{1}{a} - 1 = 1 \\
\Rightarrow a = 0.5
\]

5. Two eigen values of a 3 \(\times\) 3 real matrix \(P\) are \((2 + \sqrt{3})\) and 3. The determinant of \(P\) is \(\underline{\quad}\).

Key: (15)

Exp: Given that \(2 + \sqrt{3}\) and 3 are two Eigen values of \(3 \times 3\) real matrix is, \(2+i\) and 3 are Eigen values.

But \(2-i\) also Eigen values (\(\because\) complex roots occurs in pair only)

\[
\text{det} = \text{Product of Eigen values} \\
= (2 + i)(2 - i) \times 3 = 5 \times 3 = 15
\]
6. Consider the Boolean operator $\#$ with the following properties:

$$x \# 0 = x, \quad x \# 1 = \bar{x}, \quad x \# x = 0 \quad \text{and} \quad x \# \bar{x} = 1.$$ 

Then $x \# y$ is equivalent to

(A) $xy + \bar{y}$  
(B) $x\bar{y} + \bar{x}y$  
(C) $\bar{x}y + xy$  
(D) $xy + \bar{x} \bar{y}$

Key: (A)

7. The 16-bit 2’s complement representation of an integer is 1111 1111 1111 0101; its decimal representation is _________.

Key: (-11)

Exp:

$$\begin{align*}
\text{1111 1111 1111 0101} & \quad \text{2's complement} \quad 0000 0000 0000 1011 \\
& \quad \text{11 and 1st bit is 1.} \\
& \quad \text{So result is -11}
\end{align*}$$

8. We want to design a synchronous counter that counts the sequence 0-1-0-2-0-3 and then repeats. The minimum number of J-K flip-flops required to implement this counter is _______.

Key: (4)

9. A processor can support a maximum memory of 4 GB, where the memory is word-addressable (a word consists of two bytes). The size of the address bus of the processor is at least ________ bits.

Key: (31)

Exp: Memory size=4GB=2^{32} bytes
Word size=2 bytes

$$\text{No. of Address bits} = \frac{\text{Memory size}}{\text{Word size}} = \frac{2^{32} \text{ bytes}}{2 \text{ bytes}} = 2^{31} \Rightarrow 31 \text{ bits}$$

10. A queue is implemented using an array such that ENQUEUE and DEQUEUE operations are performed efficiently. Which one of the following statements is CORRECT (n refers to the number of items in the queue)?

(A) Both operations can be performed in $O(1)$ time

(B) At most one operation can be performed in $O(1)$ time but the worst case time for the other operation will be $\Omega(n)$
(C) The worst case time complexity for both operations will be $\Omega(n)$

(D) Worst case time complexity for both operations will be $\Omega(\log n)$

Key: (A)

11. Consider the following directed graph:

![Graph Image]

The number of different topological orderings of the vertices of the graph is ________.

Key: (6)

Exp:

- $abcdef$
- $adbecf$
- $abdecf$
- $adbecf$
- $abdecf$
- $adbecf$

12. Consider the following C program.

```c
void f(int, short);
void main()
{
    int i = 100;
    short s = 12;
    short *p = &s;
    __________;   // call to f()
}
```

Which one of the following expressions, when placed in the blank above, will NOT result in a type checking error?

(A) $f(s,*s)$  (B) $i = f(i,s)$  (C) $f(i,*s)$  (D) $f(i,*p)$
Key: (D)

Exp: Here function \( f \) takes two arguments one is int and the other is short and its return type is void. So, in main function ‘P’ is a pointer to short and when we call \( f(i,*p) \) there won’t be any type checking error.

13. The worst case running times of Insertion sort, Merge sort and Quick sort, respectively, are:

(A) \( \Theta(n \log n), \Theta(n \log n), \text{ and } \Theta(n^2) \)

(B) \( \Theta(n^2), \Theta(n^2), \text{ and } \Theta(n \log n) \)

(C) \( \Theta(n^2), \Theta(n \log n), \text{ and } \Theta(n \log n) \)

(D) \( \Theta(n^2), \Theta(n \log n), \text{ and } \Theta(n^2) \)

Key: (D)

Exp: Merge sort \( \Theta(n \log n) \) in all the cases

Quick sort \( \Theta(n \log n) \) best case and \( \Theta(n^2) \) worst cases

Insertion sort \( \Theta(n) \) best case R \( \Theta(n^2) \) worst case

14. Let \( G \) be a weighted connected undirected graph with distinct positive edge weights. If every edge weight is increased by the same value, then which of the following statements is/are TRUE?

P: Minimum spanning tree of \( G \) does not change

Q: Shortest path between any pair of vertices does not change

(A) P only \hspace{1cm} (B) Q only \hspace{1cm} (C) Neither P nor Q \hspace{1cm} (D) Both P and Q

Key: (A)

15. Consider the following C program.

```c
#include<stdio.h>

void mystery(int *ptra, int *ptrb) {
    int *temp;
    temp = ptrb;
    ptrb = ptra;
    ptra = temp;
}
```
int main() {
    int a=2016, b=0, c=4, d=42;
    mystery(&a, &b);
    if (a < c)
        mystery(&c, &a);
    mystery(&a, &d);
    printf("%d
", a);
}

The output of the program is ______.

Key: (2016)

Exp: Output is not affected by the function mystery() as it is just taking the address of a & b into ptra & ptrb and contents of ptra & ptrb are swapped leaving a & b as it is.

16. Which of the following languages is generated by the given grammar?

S → aSbS|e

(A) \{a^n b^n | n, m ≥ 0\}

(B) \{w ∈ \{a, b\}^* | w has equal number of a's and b's\}

(C) \{a^n | n ≥ 0\} ∪ \{b^n | n ≥ 0\} ∪ \{a^n b^n | n ≥ 0\}

(D) \{a, b\}^*

Key: (D)

Exp: Given grammar generates all strings of a’s and b’s including null string

∴ L = (a + b)^*

17. Which of the following decision problems are undecidable?

I. Given NFAs N_1 and N_2, is L(N_1) ∩ L(N_2) = Φ?

II. Given a CFG G = (N, Σ, P, S) and a string x ∈ Σ*, does x ∈ L(G)?

III. Given CFGs G_1 and G_2, is L(G_1) = L(G_2)?

IV. Given a TM M, is L(M) = Φ?

(A) I and IV only (B) II and III only

(C) III and IV only (D) II and IV only
Key:  (C)

Exp:  There is no known algorithm to check whether the language accepted by TM is empty. Similarly there is no algorithm to check whether language CFG’s are equivalent.

18. Which one of the following regular expressions represents the language: the set of all binary strings having two consecutive 0s and two consecutive 1s?

(A) $(0 + 1)^*0011(0 + 1)^* + (0 + 1)^*1100(0 + 1)^*$

(B) $(0 + 1)^*00(0 + 1)^*11 + 11(0 + 1)^*00(0 + 1)^*$

(C) $(0 + 1)^*00(0 + 1)^* + (0 + 1)^*11(0 + 1)^*$

(D) $00(0 + 1)^*11 + 11(0 + 1)^*00$

Key:  (B)

Exp:  (a) contains 00 & 11 consecutively which is not the required condition.

(c) Doesn’t guaranty that both 00 & 11 will be present in the string.

(d) Says string should start with 11 & ends with 00 or vice versa.

19. Consider the following code segment.

```
x = u - t;
y = x * v;
x = y + w;
y = t - z;
y = x * y;
```

The minimum number of total variables required to convert the above code segment to static single assignment form is _________.

Key:  (7)

20. Consider an arbitrary set of CPU-bound processes with unequal CPU burst lengths submitted at the same time to a computer system. Which one of the following process scheduling algorithms would minimize the average waiting time in the ready queue?

(A) Shortest remaining time first

(B) Round-robin with time quantum less than the shortest CPU burst

(C) Uniform random

(D) Highest priority first with priority proportional to CPU burst length

Key:  (A)

Exp:  SRTF is pre emptive SJF which produces less average waiting time.
21. Which of the following is NOT a super key in a relational schema with attributes V, W, X, Y, Z and primary key V Y?

(A) V XY Z (B) V W X Z (C) V W XY (D) V W XY Z

Key: (B)

Exp: Any superset of VY is a super key.

22. Which one of the following is NOT a part of the ACID properties of database transactions?

(A) Atomicity (B) Consistency (C) Isolation (D) Deadlock-freedom

Key: (D)


23. A database of research articles in a journal uses the following schema. (VOLUME, NUMBER, STARTPAGE, ENDPAGE, TITLE, YEAR, PRICE)

The primary key is (VOLUME, NUMBER, STARTPAGE, ENDPAGE) and the following functional dependencies exist in the schema.

(VOLUME, NUMBER, STARTPAGE, ENDPAGE) → TITLE
(VOLUME, NUMBER) → YEAR
(VOLUME, NUMBER, STARTPAGE, ENDPAGE) → PRICE

The database is redesigned to use the following schemas.

(VOLUME, NUMBER, STARTPAGE, ENDPAGE, TITLE, PRICE) (VOLUME, NUMBER, YEAR)

Which is the weakest normal form that the new database satisfies, but the old one does not?

(A) 1NF (B) 2NF (C) 3NF (D) BCNF

Key: (A)

Exp: candidate key is (volume, number, start page, end page)

(Volume number) → year is a partial dependency. So original table is in 1NF but not in 2NF

24. Which one of the following protocols is NOT used to resolve one form of address to another one?

(A) DNS (B) ARP (C) DHCP (D) RARP

Key: (C)
25. Which of the following is/are example(s) of stateful application layer protocols?

(i) HTTP  (ii) FTP  (iii) TCP  (iv) POP3

(A) (i) and (ii) only  
(B) (ii) and (iii) only  
(C) (ii) and (iv) only  
(D) (iv) only

Key: (C)

Exp: FTP and POP3 are stateful application layer protocols.

Q. No. 26 – 55 Carry Two Marks Each

26. The coefficient of $x^{12}$ in $(x^3 + x^4 + x^5 + x^6 + ...)^3$ is _________.

Key: (10)

Exp: 

$= x^3 (1 + x + x^2 + ...)^3$

$= x^3 (1 - x)^{-3}$

$= x^3 \sum_{n=0}^{\infty} \frac{(n+1)(n+2)}{2} x^n$

For coefficient of $x^{12}$ put $n=3 = \frac{4 \times 5}{2} = 10$

27. Consider the recurrence relation $a_1 = 8, a_n = 6n^2 + 2n + a_{n-1}$. Let $a_{99} = K \times 10^l$. The value of $K$ is _________.

Key: (198)

Exp: The recurrence relation can be written as $a_n - a_{n-1} = 6n^2 + 2n$ ... (i)

Characteristic equation is $m-1=0, m=1$

Complementary solution $= a^{(0)}_n = C_1 (1)^n = C_1$

Let the particular solution be $a^{(p)}_n = (An^2 + Bn + c)n$ ... (2)
(∵ RHS is second degree polynomial and 1 is root)

By substituting \( a_n = (An^2 + Bn + C)n \) in (1) and solving \( A=2, B=4, C=2 \)

General solution is \( a_n = a_n^{(a)} + a_n^{(b)} = C_1 + \left(2n^2 + 4n + 2\right)n \)

given \( a_i = B \Rightarrow B = c_i + B \Rightarrow c_i = 0 \)

Given \( a_{10} = k \times 10^4 \)

\[
\Rightarrow \left[2(99)^2 + 4(99) + 2\right]99 = 2\left[(100 - 1)^2 + 2(100-1) + (100-1)\right] = 10^4(198) = K \times 10^4
\]

\( K=198 \)

28. A function \( f : \mathbb{N}^+ \rightarrow \mathbb{N}^+ \), defined on the set of positive integers \( \mathbb{N}^+ \), satisfies the following properties

\[ f(n) = f\left(\frac{n}{2}\right) \quad \text{if } n \text{ is even} \]

\[ f(n) = f(n + 5) \quad \text{if } n \text{ is odd} \]

Let \( R = \{ i \mid \exists j: f(j) = i \} \) be the set of distinct values that \( f \) takes. The maximum possible size of \( R \) is ____________.

Key: (2)

Exp: Given \( f(n) = f\left(\frac{n}{2}\right) \) if \( n \) is even

\[ = f(n+5) \text{ if } n \text{ is odd} \]

We can observe that \( f(1) = f(2) = f(3) = f(4) = f(6) = f(7) \ldots \)

and \( f(5) = f(10) = f(15) = \ldots \)

Clearly, the range of \( f(x) \) will contain two distinct elements only.

29. Consider the following experiment.

Step 1. Flip a fair coin twice.

Step 2. If the outcomes are (TAILS, HEADS) then output Y and stop.

Step 3. If the outcomes are either (HEADS, HEADS) or (HEADS, TAILS), then output N and stop.

Step 4. If the outcomes are (TAILS, TAILS), then go to Step 1.

The probability that the output of the experiment is Y is (up to two decimal places) ____________.
Key:  (0.33)

Exp: From the given steps we can observe that probabilities of y are

\[
\frac{1}{4} \left( \frac{1}{4} \right) \left( \frac{1}{4} \right) \left( \frac{1}{4} \right) \frac{1}{4} \ldots
\]

Required probability

\[
\frac{1}{4} + \left( \frac{1}{4} \times \frac{1}{4} \right) + \left( \frac{1}{4} \right)^2 \frac{1}{4} + \ldots
\]

\[
= \frac{1}{4} + \left( \frac{1}{4} \right)^2 + \left( \frac{1}{4} \right)^3 + \ldots
\]

\[
= \frac{1}{4} \left( 1 + \frac{1}{4} + \left( \frac{1}{4} \right)^2 + \ldots \right) = \frac{1}{4} \times \frac{1}{1 - \frac{1}{4}} = \frac{1}{3} = 0.33
\]

30. Consider the two cascaded 2-to-1 multiplexers as shown in the figure.

The minimal sum of products form of the output X is

(A) \( \overline{P}Q + PQR \)  \quad (B) \( PQ + QR \)  \quad (C) \( PQ + \overline{P}QR \)  \quad (D) \( \overline{Q}\overline{R} + PQR \)

Key:  (D)

Exp: Output of first multiplexer is \( Y_i = \overline{P}0 + PR = PR \)

Output of second multiplexer is \( X = \overline{Q}\overline{R} + QY_i = \overline{Q}\overline{R} + QPR \Rightarrow \overline{Q}\overline{R} + PQR \)

31. The size of the data count register of a DMA controller is 16 bits. The processor needs to transfer a file of 29,154 kilobytes from disk to main memory. The memory is byte addressable. The minimum number of times the DMA controller needs to get the control of the system bus from the processor to transfer the file from the disk to main memory is ________.
32. The stage delays in a 4-stage pipeline are 800, 500, 400 and 300 picoseconds. The first stage (with delay 800 picoseconds) is replaced with a functionally equivalent design involving two stages with respective delays 600 and 350 picoseconds. The throughput increase of the pipeline is ________ percent.

Key: (33.33)

Exp:

Old design $t_p = 800$

New design $t_p = 600$

Throughput $= \frac{800 - 600}{600} \times 100\% = 33.33\%$

33. Consider a carry look ahead adder for adding two n-bit integers, built using gates of fan-in at most two. The time to perform addition using this adder is

(A) $\Theta(1)$
(B) $\Theta(\log(n))$
(C) $\Theta(\sqrt{n})$
(D) $\Theta(n)$

Key: (B)

34. The following function computes the maximum value contained in an integer array $p[ ]$ of size $n$ ($n \geq 1$).

```c
int max(int *p, int n) {
    int a=0, b=n-1;
    while (__________) {
        if (p[a] <= p[b]) { a = a+1; }
        else { b = b-1; }
    }
    return p[a];
}
```

The missing loop condition is

(A) $a \neq n$
(B) $b \neq 0$
(C) $b > (a + 1)$
(D) $b \neq a$
Key:  (D)

Exp:  When \( a=b \) then \( P[a] \) will have the maximum value of the array

35. What will be the output of the following C program?

```c
void count(int n) {
    static int d=1;
    printf("%d  ", n); printf("%d  ", d); d++;
    if(n>1) count(n-1);
    printf("%d  ", d);
}
void main() {
    count(3);
}
```

(A) 3 1 2 2 1 3 4 4 4  
(B) 3 1 2 1 1 2 2 2  
(C) 3 1 2 2 1 3 4  
(D) 3 1 2 1 1 1 2

Key:  (A)

Exp:  Output is 31 22 13 4 4 4

36. What will be the output of the following pseudo-code when parameters are passed by reference and dynamic scoping is assumed?

```plaintext
a=3;
```

Output is 31 22 13 4 4 4
void n(x) { x = x * a; print(x); }
void m(y) { a = 1; a = y - a; n(a); print(a); }
void main() { m(a); }

(A) 6, 2  (B) 6, 6  (C) 4, 2  (D) 4, 4

Key:  (D)

Exp: Dynamic scoping looks for the definition of free variable in the reverse order of calling sequence.

37. An operator delete (i) for a binary heap data structure is to be designed to delete the item in the i-th node. Assume that the heap is implemented in an array and i refers to the i-th index of the array. If the heap tree has depth d (number of edges on the path from the root to the farthest leaf), then what is the time complexity to re-fix the heap efficiently after the removal of the element?

(A) O(1)  (B) O(d) but not O(1)
(C) O(2d) but not O(d)  (D) O(d 2d) but not O(2d)

Key:  (B)

Exp: Time complexity of heapification is O (height) = O(d)

38. Consider the weighted undirected graph with 4 vertices, where the weight of edge \{i, j\} is given by the entry \(W_{ij}\) in the matrix \(W\).

\[
W = \begin{bmatrix}
0 & 2 & 8 & 5 \\
2 & 0 & 5 & 8 \\
8 & 5 & 0 & x \\
5 & 8 & x & 0
\end{bmatrix}
\]

The largest possible integer value of \(x\), for which at least one shortest path between some pair of vertices will contain the edge with weight \(x\) is __________.

Key:  (12)

Exp:

If \(x = 12\) then the shortest path between \(d\) & \(c\) will contain edge with lable ‘\(x\)’.
39. Let G be a complete undirected graph on 4 vertices, having 6 edges with weights being 1, 2, 3, 4, 5, and 6. The maximum possible weight that a minimum weight spanning tree of G can have is __________.

Key: (7)

Exp:

```
\begin{center}
\begin{tikzpicture}
  \node (A) at (0,0) {1};
  \node (B) at (1,1) {4};
  \node (C) at (1,-1) {3};
  \node (D) at (-1,1) {5};
  \node (E) at (-1,-1) {6};
  \node (F) at (2,0) {2};
  \draw (A) -- (B);
  \draw (A) -- (C);
  \draw (B) -- (D);
  \draw (C) -- (E);
  \draw (D) -- (F);
  \draw (E) -- (F);
\end{tikzpicture}
\end{center}
```

40. G = (V, E) is an undirected simple graph in which each edge has a distinct weight, and e is a particular edge of G. Which of the following statements about the minimum spanning trees (MSTs) of G is/are TRUE?

I. If e is the lightest edge of some cycle in G, then every MST of G includes e

II. If e is the heaviest edge of some cycle in G, then every MST of G excludes e

(A) I only    (B) II only   (C) both I and II    (D) neither I nor II

Key: (B)

41. Let Q denote a queue containing sixteen numbers and S be an empty stack. Head(Q) returns the element at the head of the queue Q without removing it from Q. Similarly Top(S) returns the element at the top of S without removing it from S. Consider the algorithm given below.

while Q is not Empty do
  if S is Empty OR Top(S) \leq Head(Q) then
    x := Dequeue(Q);
    Push(S, x);
  else
    x := Pop(S);
    Enqueue(Q, x);
  end
end

The maximum possible number of iterations of the while loop in the algorithm is __________.
42. Consider the following context-free grammars:
   
   G1: S → aSb, B → bB
   
   G2: S → aAbB, A → aAbB ε, B → bBε
   
   Which one of the following pairs of languages is generated by G1 and G2, respectively?
   
   (A) \{a^m b^n \mid m > 0 \text{ or } n > 0\} and \{a^m b^n \mid m > 0 \text{ and } n > 0\}
   
   (B) \{a^m b^n \mid m > 0 \text{ and } n > 0\} and \{a^m b^n \mid m > 0 \text{ and } n \leq 0\}
   
   (C) \{a^m b^n \mid m \geq 0 \text{ or } n > 0\} and \{a^m b^n \mid m > 0 \text{ and } n > 0\}
   
   (D) \{a^m b^n \mid m \geq 0 \text{ or } n > 0\} and \{a^m b^n \mid m > 0 \text{ or } n > 0\}
   
   Key: (D)

   Exp: Lagrange’s generated by \( G_1 = a^* b^+ \)
   
   Lagrange’s generated by \( G_2 = a^* b^- b^+ \)

43. Consider the transition diagram of a PDA given below with input alphabet \( \Sigma = \{a, b\} \) and stack alphabet \( \Gamma = \{X, Z\} \). Z is the initial stack symbol. Let \( L \) denote the language accepted by the PDA.

   Which one of the following is TRUE?
   
   (A) \( L = \{a^m b^n \mid m \geq 0 \} \) and is not accepted by any finite automata
   
   (B) \( L = \{a^m \mid m \geq 0 \} \cup \{a^m b^n \mid m \geq 0 \} \) and is not accepted by any deterministic PDA
   
   (C) \( L \) is not accepted by any Turing machine that halts on every input
   
   (D) \( L = \{a^m \mid m \geq 0 \} \cup \{a^m b^n \mid m \geq 0 \} \) and is deterministic context-free
   
   Key: (D)

44. Let \( X \) be a recursive language and \( Y \) be a recursively enumerable but not recursive language.

   Let \( W \) and \( Z \) be two languages such that \( Y \) reduces to \( W \), and \( Z \) reduces to \( X \) (reduction means the standard many-one reduction). Which one of the following statements is TRUE?
   
   (A) \( W \) can be recursively enumerable and \( Z \) is recursive.
45. The attributes of three arithmetic operators in some programming language are given below.

<table>
<thead>
<tr>
<th>Operator</th>
<th>Precedence</th>
<th>Associativity</th>
<th>Arity</th>
</tr>
</thead>
<tbody>
<tr>
<td>+</td>
<td>High</td>
<td>Left</td>
<td>Binary</td>
</tr>
<tr>
<td>−</td>
<td>Medium</td>
<td>Right</td>
<td>Binary</td>
</tr>
<tr>
<td>*</td>
<td>Low</td>
<td>Left</td>
<td>Binary</td>
</tr>
</tbody>
</table>

The value of the expression $2 - 5 + 1 - 7 \times 3$ in this language is $\_\_\_\_\_$. 

Key: (9)

Exp:

\[
\begin{align*}
2 - 5 + 1 - 7 \times 3 &= 2 - 5 + 1 - 7 \times 3 \\
&= 2 - 6 - 7 \times 3 \\
&= 2 - (6 - 7) \times 3 \\
&= 2 - (1) \times 3 \\
&= (2 + 1) \times 3 \\
&= 3 \times 3 = 9
\end{align*}
\]

46. Consider the following Syntax Directed Translation Scheme (SDTS), with non-terminals \{S, A\} and terminals \{a, b\}.

\[
\begin{align*}
S &\to aA \ \{\text{print } 1\} \\
S &\to a \ \{\text{print } 2\} \\
A &\to Sb \ {\text{print } 3}\end{align*}
\]

Using the above SDTS, the output printed by a bottom-up parser, for the input aab is:

(A) 1 3 2       (B) 2 2 3       (C) 2 3 1       (D) syntax error

Key: (C)

Exp:
47. Consider a computer system with 40-bit virtual addressing and page size of sixteen kilobytes. If the computer system has a one-level page table per process and each page table entry requires 48 bits, then the size of the per-process page table is _________ megabytes.

Key: (384)  
Exp: Given \( LA = 40 \text{ bit} \) \( = 2^{40} \)  
Page size \( = 16 \text{ KB} \)  
Page table Entry size \( (e) = 48 \text{ bits} \) \( = 6 \text{ bytes} \)  

Size of the page table \( = n \times e \)  
\[ \text{No. of pages} \ (n) = \frac{\text{LAS}}{PS} = \frac{2^{40}}{2^{14}} = 2^{26} = 64 \text{M} \]  
Page table size \( = 64 \times 6 \text{B} = 384 \text{MB} \)

48. Consider a disk queue with requests for I/O to blocks on cylinders 47, 38, 121, 191, 87, 11, 92, 10. The C-LOOK scheduling algorithm is used. The head is initially at cylinder number 63, moving towards larger cylinder numbers on its servicing pass. The cylinders are numbered from 0 to 199. The total head movement (in number of cylinders) incurred while servicing these requests is __________.

Key: (346)  
Exp: C-Look disc Scheduling  
\[ 0 \ 10 \ 11 \ 38 \ 47 \ 63 \ 87 \ 92 \ 121 \ 191 \ 199 \]

Total Head movements = 24 + 5 + 29 + 70 + 101 + 27 + 9 = 346
49. Consider a computer system with ten physical page frames. The system is provided with an access sequence \((a_1, a_2, \ldots, a_{20})\), where each \(a_i\) is a distinct virtual page number. The difference in the number of page faults between the last-in-first-out page replacement policy and the optimal page replacement policy is _________.

Key: (1)

Exp: \(a_1, a_2, \ldots, a_{20}, a_2, a_3, \ldots, a_{20}\)

LIFO

0 \[a_1\]
1 \[a_2\]
2 \[a_3\]
3 \[a_4\]
4 \[a_5\]
5 \[a_6\]
6 \[a_7\]
7 \[a_8\]
8 \[a_9\]
9 \(a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16}, a_{17}, a_{18}, a_{19}, a_{20}\)

For first \(a_1\) to \(a_{20}\) 20 page fault

Now \(a_1\) to \(a_9\) Hit

again \(a_{10}\) to \(a_{20}\) replace only 9th position, so 11 page fault.

So total 31 page fault

Optimal

0 \[a_1\]
1 \[a_2\]
2 \[a_3\]
3 \[a_4\]
4 \[a_5\]
5 \[a_6\]
6 \[a_7\]
7 \[a_8\]
8 \[a_9\]
9 \(a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16}, a_{17}, a_{18}, a_{19}, a_{20}\)

For first \(a_1\) to \(a_{20}\) 20 fault

Next \(a_1\) to \(a_9\) Hit

again \(a_{10}\) to \(a_{19}\) replace any location from 0 to 9 for \(a_{20}\) Hit.

So total 30 page fault

Difference \(= 31 - 30 = 1\)
50. Consider the following proposed solution for the critical section problem. There are \( n \) processes: \( P_0, \ldots, P_{n-1} \). In the code, function \( \text{pmax} \) returns an integer not smaller than any of its arguments. For all \( i \), \( t[i] \) is initialized to zero.

Code for \( P_i \):

\[
do \{
    c[i]=1; t[i] = \text{pmax}(t[0], \ldots, t[n-1])+1; \ c[i]=0;
    \text{for every } j = i \text{ in } \{0, \ldots, n-1\} \{
        \text{while } (c[j]);
        \text{while } (t[j] != 0 \text{ && } t[j] \leq t[i]);
    \}
    \text{Critical Section;}
    t[i]=0;
    \text{Remainder Section;}
\} \text{ while (true);}\
\]

Which one of the following is TRUE about the above solution?

(A) At most one process can be in the critical section at any time
(B) The bounded wait condition is satisfied
(C) The progress condition is satisfied
(D) It cannot cause a deadlock

Key: (A)

51. Consider the following two phase locking protocol. Suppose a transaction \( T \) accesses (for read or write operations), a certain set of objects \( \{O_1, \ldots, O_k\} \). This is done in the following manner:

Step 1. \( T \) acquires exclusive locks to \( O_1, \ldots, O_k \) in increasing order of their addresses.

Step 2. The required operations are performed.

Step 3. All locks are released.

This protocol will

(A) guarantee serializability and deadlock-freedom
(B) guarantee neither serializability nor deadlock-freedom
(C) guarantee serializability but not deadlock-freedom
(D) guarantee deadlock-freedom but not serializability
Key: (A)

Exp: 2PL ensures serializability and here as we are following linear order in acquiring the locks there will not be any deadlock.

52. Consider that B wants to send a message m that is digitally signed to A. Let the pair of private and public keys for A and B be denoted by $K^-_A$ and $K^+_B$ for $x = A, B$, respectively. Let $K_x(m)$ represent the operation of encrypting m with a key $K_x$ and $H(m)$ represent the message digest. Which one of the following indicates the CORRECT way of sending the message m along with the digital signature to A?

(A) $\{m, K^+_B(H(m))\}$

(B) $\{m, K^-_B(H(m))\}$

(C) $\{m, K^-_A(H(m))\}$

(D) $\{m, K^+_A(m)\}$

Key: (B)

Exp:

53. An IP datagram of size 1000 bytes arrives at a router. The router has to forward this packet on a link whose MTU (maximum transmission unit) is 100 bytes. Assume that the size of the IP header is 20 bytes.

The number of fragments that the IP datagram will be divided into for transmission is ________.

Key: (13)

Exp:

So, no. of fragments that are transferred in this scenario is 13.
54. For a host machine that uses the token bucket algorithm for congestion control, the token bucket has a capacity of 1 megabyte and the maximum output rate is 20 megabytes per second. Tokens arrive at a rate to sustain output at a rate of 10 megabytes per second. The token bucket is currently full and the machine needs to send 12 megabytes of data. The minimum time required to transmit the data is __________ seconds.

Key: \((1.2)\)

Exp: Given

\[
C = 1 \text{Mb} \\
\text{Max Output rate} = 20 \text{Mbps} \\
\text{Arrival rate} = 10 \text{Mbps}
\]

\[
\therefore \text{The minimum time required to transmit the data is } S = \frac{c}{m - \rho}
\]

\[
S = \frac{1 \text{Mb}}{20 - 10 \text{Mbps}} = \frac{1}{10} = 0.1 \text{ sec}
\]

For 12Mb of data, S value becomes 1.2 seconds

55. A sender uses the Stop-and-Wait ARQ protocol for reliable transmission of frames. Frames are of size 1000 bytes and the transmission rate at the sender is 80 Kbps \((1 \text{Kbps} = 1000 \text{ bits/second})\). Size of an acknowledgement is 100 bytes and the transmission rate at the receiver is 8 Kbps. The one-way propagation delay is 100 milliseconds.

Assuming no frame is lost, the sender throughput is __________ bytes/second.

Key: \((2500)\)

Exp: Frame size \((L) = 1000 \text{bytes}\)

Sender side bandwidth \((B_S) = 80 \text{kbps}\)

Acknowledgement \((L_A) = 100 \text{bytes}\)

Receiver side bandwidth \((B_R) = 8 \text{kbps}\)

\[
T_p = 100 \text{ms}
\]

\[
n = \frac{T_s}{T_s + T_{ack} + 2T_p}
\]

\[
(\text{msg}) T_s = \frac{L}{B_S} = \frac{1000 \text{Bytes}}{10 \times 10^6 \text{ BPS}} = 100 \text{ms}
\]

\[
(Ack) T_A = \frac{L_A}{B_R} = \frac{100 \text{Bytes}}{1 \times 10^7 \text{BPS}} = 100 \text{ms}
\]

\[
T_p = 100 \text{ms}
\]
\[
\therefore \text{Channel Utilization} = \frac{T_a}{T_a + T_{\text{ack}} + 2T_p} = \frac{100\text{ms}}{100\text{ms} + 100\text{ms} + 200\text{ms}} = \frac{1}{4}
\]

\[
\therefore \text{Throughput} = \eta \times B = \frac{1}{4} \times 10 \times 10^3 = 2.5 \text{ Kbps (or 2500 Bps)}
\]