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Section-I: General Ability

1. Arrange the following three-dimensional objects in the descending order of their volumes:
   (i) A cuboid with dimensions 10 cm, 8 cm and 6 cm
   (ii) A cube of side 8 cm
   (iii) A cylinder with base radius 7 cm and height 7 cm
   (iv) A sphere of radius 7 cm
   (A) (i), (ii), (iii), (iv)  
   (B) (ii), (i), (iv), (iii)  
   (C) (iii), (ii), (i), (iv)  
   (D) (iv), (iii), (ii), (i)

Key: (D)

Exp: i. Volume of cuboid = \( \ell \times b \times h = 10 \times 8 \times 6 = 480 \text{cm}^3 \)
   ii. Volume of cube = \( a^3 = 8^3 = 512 \text{cm}^3 \)
   iii. Volume of cylinder = \( \pi r^2 h = \pi (7)^2 \times 7 = 343\pi \approx 1077.57 \)
   iv. Volume of sphere = \( \frac{4}{3} \pi r^3 = \frac{4}{3} \pi (7)^3 \approx 1436.76 \)

2. For \( 0 \leq x \leq 2\pi \), \( \sin x \) and \( \cos x \) are both decreasing functions in the interval _____.

   (A) \( 0, \frac{\pi}{2} \)  
   (B) \( 0, \pi \)  
   (C) \( \frac{3\pi}{2}, \frac{5\pi}{2} \)  
   (D) \( \frac{3\pi}{2}, 2\pi \)

Key: (B)

Exp: \( \sin x \) is increasing & \( \cos x \) is decreasing

In \( 0, \frac{\pi}{2} \), \( \cos x \) is decreasing & \( \sin x \) is increasing

In \( \frac{\pi}{2}, \pi \), \( \cos x \) is decreasing & \( \sin x \) is increasing

In \( \pi, \frac{3\pi}{2} \), \( \cos x \) is increasing & \( \sin x \) is decreasing

In \( \frac{3\pi}{2}, 2\pi \), \( \cos x \) is increasing & \( \sin x \) also increasing
3. “When she fell down the _____, she received many _____ but little help.”
   The words that best fill the blanks in the above sentence are
   (A) stairs, stares (B) stairs, stairs
   (C) stares, stairs (D) stares, stares
   Key: (A)

4. “In spite of being warned repeatedly, he failed to correct his ____ behaviour.”
   The word that best fills the blank in the above sentence is
   (A) rational (B) reasonable (C) errant (D) good
   Key: (C)

5. The area of an equilateral triangle is $\sqrt{3}$. What is the perimeter of the triangle?
   (A) 2 (B) 4 (C) 6 (D) 8
   Key: (C)
   Exp: Given $\frac{\sqrt{3}}{4}a^2 = \sqrt{3}$ where $a$ is side of an equilateral triangle
   \[ \Rightarrow a^2 = 4 \]
   \[ \Rightarrow a = 2 \]
   Perimeter $= 3 \times 2 = 6$

6. An automobile travels from city A to city B and returns to city A by the same route. The speed of
   the vehicle during the onward and return journeys were constant at 60 km/h and 90 km/h,
   respectively. What is the average in km/h for the entire journey?
   (A) 72 (B) 73 (C) 74 (D) 75
   Key: (A)
   Exp: In this case, Avg speed $= \frac{2(60 \times 90)}{(60 + 90)} = 72$

7. In a detailed study of annual crow births in India, it was found that there was relatively no growth
   during the period 2002 to 2004 and a sudden spike from 2004 to 2005. In another unrelated study, it
   was found that the revenue from cracker sales in India which remained fairly flat from 2002 to
   2004, saw a sudden spike in 2005 before declining again in 2006. The solid line in the graph below
   refers to annual sale of crackers and the dashed line refers to the annual crow births in India.
   Choose the most appropriate inference from the above data.
(A) There is a strong correlation between crow birth and cracker sales.
(B) Cracker usage increases crow birth rate.
(C) If cracker sale declines, crow birth will decline.
(D) Increased birth rate of crows will cause an increase in the sale of crackers.

Key: (A)

Exp: By graph, we can say that both the cracker sales and crow births are proportional.
∴ There is a strong correlation between crow births and cracker sales.

8. To pass a test, a candidate needs to answer at least 2 out of 3 questions correctly. A total of 6,30,000 candidates appeared for the test. Question A was correctly answered by 3,30,000 candidates. Question B was answered correctly by 2,50,000 candidates. Question C was answered correctly by 2,60,000 candidates. Both questions A and B were answered correctly by 1,00,000 candidates. Both questions B and C were answered correctly by 90,000 candidates. Both questions A and C were answered correctly by 80,000 candidates. If the number of students answering all questions correctly is the same as the number answering none, how many candidates failed to clear the test?

(A) 30,000  (B) 2,70,000  (C) 3,90,000  (D) 4,20,000

Key: (D)

Exp: Let no. of candidates answering all questions correctly = no. of candidates answering none = x

Total = 6,30,000

A = 3,30,000
B = 2,50,000

\[\begin{align*}
\text{A} & = 3,30,000 \\
\text{B} & = 2,50,000 \\
\text{C} & = 2,60,000 \\
\text{p} & = 100,000 - x \\
\text{q} & = 90,000 \\
\text{r} & = 80,000 \\
\text{x} & = \text{(None)}
\end{align*}\]
\[ p + (1,00,000 - x) + x + (80,000 - x) = 3,30,000 \]
\[ \Rightarrow p = 1,50,000 + x \]
\[ q + (1,00,000 - x) + x + (90,000 - x) = 2,50,000 \]
\[ \Rightarrow q = 60000 + x \]
\[ r + (80,000 - x) + x + (90,000 - x) = 2,60,000 \]
\[ \Rightarrow r = 90,000 + x \]

\[ 6,30,000 = (1,50,000 + x) + (60,000 + x) + (90,000 + x) \]
\[ + (1,00,000 - x) + (90,000 - x) + (80,000 - x) + x + x \]

\[ 6,30,000 = 5,70,000 + 2x \]
\[ \Rightarrow x = 30,000 \]

\[ \therefore \text{No. of candidates answered at least two questions} \]
\[ = (1,00,000 - x) + (90,000 - x) + (80,000 - x) + x \]
\[ = 2,10,000 \]

Candidates failed to clear the test
\[ = \text{Total} - \text{candidates passed the test} \]
\[ = \text{Total} - \text{No. of candidates answered at least two questions} \]
\[ = 6,30,000 - 2,10,000 \]
\[ = 4,20,000 \]

9. A set of 4 parallel lines intersect with another set of 5 parallel lines. How many parallelograms are formed?

(A) 20  (B) 48  (C) 60  (D) 72

Key: (C)

Exp: In this case,

Number of parallelograms formed \[= \binom{4}{2} \times \binom{5}{2} \]
\[= 6 \times 10 \]
\[= 60 \]

10. If \[x^2 + x - 1 = 0 \] what is the value of \[x^4 + \frac{1}{x^2}\]?

(A) 1  (B) 5  (C) 7  (D) 9

Key: (C)

Exp: Given \[x^2 + x - 1 = 0 \]
\[ x^2 + x = 1 \Rightarrow x(x + 1) = 1 \]
\[ x + 1 = \frac{1}{x} \]
\[ x - \frac{1}{x} = -1 \]

Squaring on both sides
\[ x^2 + \frac{1}{x^2} - 2 = 1 \Rightarrow x^2 + \frac{1}{x^2} = 3 \]

Again squaring on both sides
\[ x^4 + \frac{1}{x^4} + 2 = 9 \]
\[ x^4 + \frac{1}{x^4} = 7 \]

Section-II: Instrumentation Engineering

1. An optical pulse containing \(6 \times 10^6\) photons is incident on a photodiode and \(4.5 \times 10^6\) electron-hole pairs are created. The maximum possible quantum efficiency (in %) of the photodiode is ____.

Key: (75)

Exp: It is given that
- Number of photon = \(6 \times 10^6\)
- Number of EHP = \(4.5 \times 10^6\)

% Quantum Efficiency (\(\eta\)) = \(\frac{Number\ of\ EHP}{Number\ of\ Photon} \times 100\)

\[
\frac{4.5 \times 10^6}{6 \times 10^6} \times 100 = 75\%
\]

2. The number of comparators required for implementing an 8-bit flash analog-to-digital converter is

(A) 8 \hspace{1cm} (B) 128 \hspace{1cm} (C) 255 \hspace{1cm} (D) 256

Key: (C)

Exp: In a \(n\) bit Flash type ADC, the number of comparator required is given by \(2^n - 1\).

\[ \Rightarrow \text{So for an 8-bit flash type ADC, the number of comparator needed is} \]
\[ 2^8 - 1 = 255 \]
3. A coil having an impedance of \((10 + j100) \Omega\) is connected in parallel to a variable capacitor as shown in figure. Keeping the excitation frequency unchanged, the value of the capacitor is changed so that parallel resonance occurs. The impedance across terminals p-q at resonance (in \(\Omega\)) is ____.

Key: (1010)

Exp: \(\rightarrow\) It is given that the circuit is under resonance, and we know at this resonant frequency
\[
\text{Img}[Y_{eq}] = 0 \text{ or Img}[Z_{eq}] = 0
\]
In a parallel circuit, from calculation point of view, it is wised to deal with \(\text{Img}[Y_{eq}] = 0\)

\(\rightarrow\) From the given circuit we can say
\[
Y_{eq} = \frac{1}{Z_{RL}} + \frac{1}{Z_C}
\]
\[
= \frac{1}{R + j\omega L} + \frac{1}{\frac{1}{j\omega C}}
\]
\[
= \frac{R - j\omega L}{R^2 + (\omega L)^2} + j\omega C
\]
\[
= \left[\frac{R}{R^2 + (\omega L)^2}\right] + j\left[\omega C - \frac{\omega L}{R^2 + (\omega L)^2}\right]
\]
at resonance frequency \(\text{Img}[Y_{eq}] = 0\), So
\[
Y_{eq} = \frac{R}{R^2 + (\omega L)^2}
\]
\[
Z_{eq} = \frac{1}{Y_{eq}} = \frac{R^2 + (\omega L)^2}{R}
\]
\[
= \frac{10^2 + 100^2}{10} = \frac{10100}{10} = 1010
\]

[From the circuit we can see the value of R and \(\omega L\)]
4. A 300 V, 5 A, 0.2 pf low power factor wattmeter is used to measure the power consumed by a load. The wattmeter scale has 150 divisions and the pointer is on the 100\textsuperscript{th} division. The power consumed by the load (in Watts) is____.

**Key: (200)**

**Exp:** → The specifications of the wattmeter is given as 300 V, 5 A, 0.2 p.f. So its full scale reading is 300\times5 \times 0.2 = 300 watts.

→ It is also given that there are 150 divisions; by default we assume the scale as uniform scale so

\[ \frac{150\text{ division}}{300\text{ watt}} \Rightarrow 1\text{ division} = \frac{300}{150} = 2\text{ watt} \]

\[ 100\text{th division} \Rightarrow 2 \times 100 = 200 \text{ watts} \]

5. The approximate phase response of \( \frac{100^2e^{-0.01s}}{s^2 + 0.2s + 100^2} \) is

**Key: (A)**

**Exp:** \( G(s) = \frac{10^4 e^{-0.01s}}{s^2 + 0.2s + 100^2} \)

Put \( s = j\omega \)

\( G(j\omega) = \frac{10^4 e^{-0.1j\omega}}{-\omega^2 + 0.2j\omega + 10^4} \)
\[ \angle G(j \omega) = -0.01\omega - \tan^{-1}\left( \frac{0.2\omega}{10^3 - \omega^2} \right) \]

\[ \omega = 0; \ \angle G(j \omega) = 0; \ \omega = 10; \ \angle G(j \omega) = -5.8^\circ \]

\[ \omega = 100; \ \angle G(j \omega) = -1 - \tan^{-1}\left( \frac{20}{0} \right) \]

\[ = - \frac{180}{\pi} - 90^\circ = -57.3 - 90^\circ = -147.3^\circ \]

\[ \omega = 200 \]

\[ \angle G(j \omega) = -\frac{360^\circ}{\pi} - \tan^{-1}\left( \frac{40}{-3 \times 10^4} \right) \]

So, Phase decreases further.

Options (A) satisfy.

6. An input \( p(t) = \sin(t) \) is applied to the system \( G(s) = \frac{s-1}{s+1} \). The corresponding steady state output is \( y(t) = \sin(t + \phi) \), where the phase \( \phi \) (in degrees), when restricted to \( 0^\circ \leq \phi \leq 360^\circ \), is ____.

**Key:** (90)

**Exp:** Given input \( p(t) = \sin(t) \)

\[ G(s) = \frac{s-1}{s+1} \quad \text{...(1)} \]

\[ y(t) = |G(j \omega)|_{\omega=1} \sin(t + \phi) \]

Where \( \phi = \angle G(j \omega) \mid_{\omega=1} \]

From (1) \( G(j \omega) = \frac{j \omega - 1}{j \omega + 1} \)

\[ \angle G(j \omega) = 180 - \tan^{-1}(\omega) - \tan^{-1}(\omega) \text{ mo} \]

\[ = 180 - 2 \tan^{-1}(\omega) \]

\[ \phi = \angle G(j \omega) \mid_{\omega=1} = 180 - 90^\circ = 90^\circ \]

\[ |G(j \omega)|_{\omega=1} = 1 \]

So, \( y(t) = 1. \sin(t + 90^\circ) \)

\( \phi = 90^\circ \)
7. A piezoelectric pressure sensor has a bandpass characteristic with cut-off frequencies of 0.1 Hz and 1 MHz, and a sensitivity of 100 mV/ kPa. The sensor is subjected to a static constant pressure of 100 kPa. Its steady-state output will be

(A) 0V    (B) 0.1 V    (C) 1V    (D) 10 V

Key: (A)

Exp: → Piezoelectric sensor always produces zero output, for a constant or static input.
→ If and only if the input is changing we get non-zero output.
→ Since the given input is static constant pressure of 100 kPa its output is 0V.

8. Consider signal \( x(t) = \begin{cases} 1, & |t| \leq 2 \\ 0, & |t| > 2 \end{cases} \). Let \( \delta(t) \) denote the unit impulse (Dirac-delta) function. The value of the integral \( \int_{0}^{5} 2x(t-3)\delta(t-4)dt \) is

(A) 2    (B) 1    (C) 0    (D) 3

Key: (A)

Exp: Given, \( x(t) = \begin{cases} 1, & |t| \leq 2 \\ 0, & |t| > 2 \end{cases} \)

\[ 2x(t-3) = \begin{cases} 2, & t = 5 \\ 0, & t \neq 5 \end{cases} \]

\[ \int_{0}^{5} 2x(t-3)\delta(t-4)dt = \int_{0}^{5} 2\cdot 1\cdot \delta(t-4)dt = 2 \]

9. Consider two functions \( f(x) = (x - 2)^2 \) and \( g(x) = 2x - 1 \), where \( x \) is real. The smallest value of \( x \) for which \( f(x) = g(x) \) is_____.

Key: 1

Exp: Given \( f(x) = (x - 2)^2 \) \& \( g(x) = 2x - 1 \)
\[ \therefore f(x) = g(x) \]
\[ \Rightarrow (x - 2)^2 = 2x - 1 \]
\[ \Rightarrow x^2 + 4 - 4x = 2x - 1 \]
\[ \Rightarrow x^2 - 6x + 5 = 0 \]
\[ \Rightarrow x^2 - 5x - x + 5 = 0 \]
\[ \Rightarrow x(x - 5) - 1(x - 5) = 0 \]
\[ \Rightarrow (x - 5)(x - 1) = 0 \Rightarrow x = 5, 1 \]

\[ \therefore \text{The smallest value of } x \text{ for which } f(x) = g(x) \text{ is } 1. \]

10. A voltage of \(6 \cos(100\pi t)\) V is fed as y-input to a CRO. The waveform seen on the screen of the CRO is shown in the figure. The Y and X axes setting for the CRO are respectively.

![Waveform Image]

(A) 1V/div and 1ms/div  
(B) 1 V/div and 2 ms/div  
(C) 2V/div and 1 ms/div  
(D) 2 V/div and 2 ms/div

**Key: (D)**

**Exp:**  
→ The input is \(6 \cos(100\pi t)\), so, it is having

Peak amplitude = 6V

Time period = \(\frac{2\pi}{100\pi} = 20\) m sec

→ From the CRO screen we can notice that peak amplitude 6V is represented by 3 Full divisions on the Y axis.

3 Full division = 6V
1 Full division = 2V

So the Y axis setting is 2V/div.

→ Similarly,

Time period 20 m. sec is represented by 10 full divisions on the X axis.
10 Full division = 20 m.sec
1 Full division = 2 m.sec

So the X axis setting is 2 m.sec/div.
11. Let \( f_1(z) = z^2 \) and \( f_2(z) = \overline{z} \) be two complex variable functions. Here \( \overline{z} \) is the complex conjugate of \( z \). Choose the correct answer.

(A) Both \( f_1(z) \) and \( f_2(z) \) are analytic
(B) Only \( f_1(z) \) is analytic
(C) Only \( f_2(z) \) is analytic
(D) Both \( f_1(z) \) and \( f_2(z) \) are not analytic

**Key:** (B)

**Exp:** Given \( f_1(z) = z^2 \) and \( f_2(z) = \overline{z} \)

Clearly, \( f_1(z) = z^2 \) is polynomial in 'z'.

\[ f_1(z) = z^2 \Rightarrow f_1(z) \text{ is analytic everywhere in the complex plane} \]

\[ f_2(z) = \overline{z} \Rightarrow f_2(z) = x - iy \quad \therefore \text{If } z = x + iy \text{ then } \overline{z} = x - iy \]

Let \( u = x; v = -y \)

\[ \frac{\partial u}{\partial x} = 1 & \frac{\partial u}{\partial y} = 0; \quad \frac{\partial v}{\partial x} = 0 & \frac{\partial v}{\partial y} = -1 \]

\[ \therefore \frac{\partial u}{\partial x} \neq \frac{\partial v}{\partial y} \]

\[ \therefore \text{Cauchy – Riemann equation failed} \]

\[ \therefore f_2(z) = \overline{z} \text{ is not analytic.} \]

12. In the given circuit, assume the opamp is ideal and the transistor has a \( \beta \) of 20.

![Circuit Diagram]

The current \( I_o \) (in \( \mu A \)) flowing through the load \( Z_L \) is _______.

**Key:** (300)

**Exp:** Given \( \beta = 20 \)
\[ I_c = \frac{5 - 2}{10.5k} = \frac{3}{10.5k}; \beta = 20 \]
\[ I_o = I_E = \frac{(1+\beta)}{\beta} I_c = 0.3\text{mA} = 300\mu\text{A} \]

Answer is 300.

13. An amplitude modulated signal is shown in the figure. The modulation index is (up to one decimal place) _____.

**Key:** (0.3)

**Exp:** In such cases,

\[
\text{Modulation index} = \frac{A(t)_{\text{max}} - A(t)_{\text{min}}}{A(t)_{\text{max}} + A(t)_{\text{min}}}
\]

Where \( A(t) \) is envelope of given waveform
\[
= \frac{13 - 7}{13 + 7} = \frac{6}{20} = \frac{3}{10} = 0.3
\]
14. Let $N$ be a $3 \times 3$ matrix with real number entries. The matrix $N$ is such that $N^2 = 0$. The eigen values of $N$ are 
(A) $0, 0, 0$ 
(B) $0, 0, 1$ 
(C) $0, 1, 1$ 
(D) $1, 1, 1$

**Key:** (A)

**Exp:** Given that:

$N$ be a $3 \times 3$ matrix with real number entries such that $N^2 = 0$.

$\Rightarrow$ $N$ is nilpotent matrix of order ‘3’

$\Rightarrow$ The eigen values of nilpotent matrix are only $0, 0, 0$.

15. Two periodic signals $x(t)$ and $y(t)$ have the same fundamental period of 3 seconds. Consider the signal $z(t) = x(-t) + y(2t + 1)$. The fundamental period of $z(t)$ in seconds is 
(A) $1$ 
(B) $1.5$ 
(C) $2$ 
(D) $3$

**Key:** (D)

**Exp:** $x(t) \rightarrow T = 3$ seconds

$x(-t) \rightarrow T = 3$ seconds

Time period does not change due to time reversal

$y(t) \rightarrow T = 3$ seconds

$y(t + 1) \rightarrow T = 3$ seconds

Due to shifting time period does not change

$y(2t + 1) \rightarrow T = 1.5$ seconds

Signal will get compress by factor of 2.

Thus $x(-t) + y(2t + 1)$ will have time period $T = \text{LCM}(1.5, 3) = 3$

16. The representation of the decimal number $(27.625)$ in base-$2$ number system is 
(A) $11011.110$ 
(B) $11101.101$ 
(C) $11011.101$ 
(D) $10111.110$

**Key:** (C)

**Exp:** $(27.625)_{10} = (?)_2$

$\rightarrow$ Integer part
\[ 2|27 \\
2|13 \rightarrow 1 \\
2|6 \rightarrow 1 \\
2|3 \rightarrow 0 \\
1 \rightarrow 1 \]

\[ \Rightarrow (27)_{10} = (11011)_{2} \]

\[ \rightarrow \] Fractional part

\[ 0.625 \times 2 = 1.250 \Rightarrow 1 \]
\[ 0.250 \times 2 = 0.500 \Rightarrow 0 \]
\[ 0.500 \times 2 = 1.000 \Rightarrow 1 \]
\[ 0.000 \times 2 = 0.000 \Rightarrow 0 \]

\[ \Rightarrow (0.625)_{10} = (0.101)_{2} \]

\[ \Rightarrow (27.625)_{10} = (11011.101)_{2} \]

17. A ideal square wave with period of 20 ms shown in the figure, is passed through an ideal low pass filter with cut-off frequency 120 Hz. Which of the following is an accurate description of the output?

(A) Output is zero

(B) Output consists of both 50 Hz and 100 Hz frequency components

(C) Output is a pure sinusoid of frequency 50 Hz.

(D) Output is a square wave of fundamental frequency of 50 Hz.

Key: (C)

Exp: Given square wave passed half wave symmetry
Thus only odd frequency compound exist.
The frequency response of filter is

![Diagram showing frequency response with H(f) and f axes]

At the output of filter only one frequency component exist.
Thus output will have pure sinusoidal of 50 Hz.

18. As shown in the figure, temperature $\theta$ is measured using a K type thermocouple. It has a sensitivity of $40 \mu\text{V/}^\circ\text{C}$. The gain (G) of the ideal instrumentation amplifier is 25. If the output $V_o$ is 96 mV, then the value of $\theta$ (in $^\circ\text{C}$) is____.

![Diagram showing temperature measurement with IN A, G = 25, and V_o]

**Key:** (96)

**Exp:** → From the given figure we can write

\[ V = (\theta - \theta_{ref}) \times \text{Sensitivity} \times \text{Gain of amplifier} \]

\[ \Rightarrow 96 \text{ mV} = (\theta - 0^\circ)(40 \mu\text{V/}^\circ\text{C})(25) \]

\[ \Rightarrow \theta = \frac{96 \times 10^{-3}}{25 \times 40 \times 10^{-6}} = 96^\circ\text{C} \]

19. Consider the transfer function $G(s) = \frac{2}{(s+1)(s+2)}$. The phase margin of $G(s)$ in degrees is ____.

**Key:** (180)

**Exp:** $G(s) = \frac{2}{(s+1)(s+2)}$

Phase margin: $= 180^\circ + \angle G(j\omega)|_{\omega = \omega_{gc}}$
20. The diodes given in the circuit are ideal. At \( t = 60 \text{ ms} \), \( V_{pq} \) (in Volts) is____.

Key: (200)

Exp: (1) During positive cycle \( D_2 \) and \( D_4 \) are OFF, and \( D_1 \) and \( D_3 \) are ON.

(2) During Negative cycle \( D_1 \) and \( D_3 \) OFF and \([C_1 \ C_2]\) capacitors. Charges up to 100 initially.

Already charged capacitors do not discharge because no resistor, and also \( t = 60 \text{ msec} \) also does not effect. On capacitor voltages

By K.V.L \( V_{pq} - V_i = 200 \)

\[ V_{pq} - 100 \sin(1080^\circ) = 200 \]

\[ V_{pq} = 200 \]

21. The Thevenin equivalent circuit representation across terminals \( p-q \) of the circuit shown in the figure is a
(A) 1 V source in series with 150 kΩ
(B) 1 V source in parallel with 100 kΩ
(C) 2 V source in series with 150 kΩ
(D) 2 V source in parallel with 200 kΩ

Key: (C)

Exp: → Thevenin voltage is the open circuit voltage across the defined terminal PQ

\[ V_{th} = \left( \frac{100}{100 + 100} \right) \times 4 \] (By voltage division)

= 2 V

→ Thevenin resistance is obtained by making all the independent source value as 0V, i.e., independent ideal voltage source (4V in this case) should be replaced by short circuit.

\[ R_{th} = 100 + [100 || 100] \]

= 100 + 50
= 150 kΩ

→ \( V_{th} = 2V \)
\( R_{th} = 150kΩ \)
22. A series R-C circuit is excited by a $10V$ sinusoidal ac voltage source. The locus diagram of the phasor current $\vec{I} = (x + jy) \, A$, when C is varied, while keeping R fixed, is

Key: (A)

Exp: We want the locus of $\vec{I}$, So its expression is

$$\vec{I} = \frac{10}{R + \frac{1}{j\omega C}} = \frac{1}{\sqrt{R^2 + (\frac{1}{\omega C})^2}} \tan^{-1}\left(\frac{1}{\omega RC}\right)$$

→ In the above equation C is variable

When $c = 0$, $\vec{I} = 0\, 90^\circ$

When $c = \infty$, $\vec{I} = \frac{1}{R} \, 0^\circ$

So the locus diagram should start with $90^\circ$ axis with magnitude 0, and should end with $0^\circ$ axis with magnitude $\frac{1}{R}$. 
23. For the 3-bit binary counter shown in the figure, the output increments at every positive transition in the clock (CLK). Assume ideal diodes and the starting state of the counter as 000. If output high is 1 V and output low is 0 V, the current I (in mA) flowing through the 50Ω resistor during the 5th clock cycle is (up to one decimal place) ____.

Key: (10)

Exp: → From the information given it is clear that the given counter is a binary up counter, if its initial state is 000, then after 5 clocks its state will be 101.

→ Since logic 1 represents 1V and logic 0 represents 0 V, the equivalent circuit can be drawn as

From the orientation of the ideal diodes and the voltage being connected to them we can say all the 3 diodes are in forward bias and hence can be replaced by short circuit as follows
By writing nodal equation at node X, we have
\[
\frac{V_x}{100} + \frac{V_x}{50} + \frac{V_x}{100} = 0 \\
\Rightarrow V_x = 0.5V \\
\Rightarrow I = \frac{V_x}{50} = (0.5) = 10mA
\]

24. X and Y are two independent random variables with variance 1 and 2, respectively. Let Z = X - Y. The variance of Z is

(A) 0 (B) 1 (C) 2 (D) 3

Key: (D)

Exp: Given, X and Y are two independent random variables with variances 1 and 2 respectively.

i.e, \( \text{var}(x) = 1 \) & \( \text{var}(y) = 2 \)

Let \( Z = x - y \)

\[ \text{var}(z) = \text{var}(x - y) = \text{var}(x) + \text{var}(y); \text{since if } x, y \text{ are two independent R.V's} \]

then \( \text{var}(ax \pm by) = a^2 \text{var}(x) + b^2 \text{var}(y) \).

\[ \Rightarrow \text{var}(z) = 1 + 2 = 3 \]

\[ \Rightarrow \text{var}(z) = 3. \]

25. Consider a sequence of tossing of a fair coin where the outcomes of tosses are independent. The probability of getting the head for the third time in the fifth toss is

(A) \( \frac{5}{16} \) (B) \( \frac{3}{16} \) (C) \( \frac{3}{5} \) (D) \( \frac{9}{16} \)

Key: (B)

Exp: Probability of getting the head for the third time in the fifth toss = probability of getting two heads out of 4 tosses and one head in 5th toss.

\[ = 4c_3 \times \left( \frac{1}{2} \right)^2 \times \left( \frac{1}{2} \right)^2 \times \frac{1}{2}; \]

\[ \begin{align*}
&= 4 \times 3 \times \frac{1}{16} \times \frac{1}{2} \\
&\Rightarrow \text{Required probability} = \frac{3}{16}.
\end{align*} \]
26. A 1000Ω strain gage (R_g) has a gage factor of 2.5. It is connected in the bridge as shown in figure. The strain gage is subjected with a positive strain of 400 μm/m. The output V_0 (in mV) of the bridge is (up to two decimal places) ____.

Key: (0.5)

Exp: → The given circuit is standard “Quarter bridge”

Whose output is given as

\[ V_0 = \frac{V_s \cdot (\Delta R)}{4R_g} \]

R_g : Strain gauge resistance
\( \Delta R \) : Charge in resistance
\( \Delta R = R \times \text{Gauge factor} \times \text{Strain} \)
\[ = R \times 2.5 \times 400 \times 10^{-6} = 1 \]
\[ \Rightarrow V_0 = \frac{V_s \cdot (\Delta R)}{4R_g} \]
\[ = \frac{2}{4 \times 1000} \times 1 = 0.5 \text{ mV} \]

27. In the given circuit, the mesh currents I_1, I_2 and I_3 are

(A) I_1 = 1A, I_2 = 2A and I_3 = 3A
(B) I_1 = 2A, I_2 = 3A and I_3 = 4A
(C) I_1 = 3A, I_2 = 4A and I_3 = 5A
(D) I_1 = 4A, I_2 = 5A and I_3 = 6A

Key: (A)

Exp:
Writing the KVL equations

→ In outer most loop through 5V
   
   \[-2I_1 + 5 - I_3 = 0\]
   
   \[\Rightarrow 2I_1 + I_3 = 5 \quad \text{....eq (1)}\]

→ In the loop containing the current source
   
   \[-2I_1 - 6(I_1 - I_2) - 1(I_3 - I_2) - 1(I_3) = 0\]
   
   \[\Rightarrow -2I_1 - 6I_1 + 6I_2 - I_3 + I_2 - I_3 = 0\]
   
   \[\Rightarrow -8I_1 + 7I_2 - 2I_3 = 0 \quad \text{.... eq(2)}\]

→ By writing KCL at node X, we have
   
   \[I_3 = I_1 + 2\]
   
   \[\Rightarrow -I_1 + I_3 = 2 \quad \text{...eq (3)}\]

→ By solving the above 3 equations, or by verifying each options to satisfy the above 3 equation,
   
   We have
   
   \[I_1 = 1A, I_2 = 2A, I_3 = 3A\]

28. The voltage and current drawn by a resistive load are measured with a 300 V voltmeter of accuracy of \(\pm 1\%\) of full scale and a 5 A ammeter of accuracy \(\pm 0.5\%\) of full scale. The readings obtained are 200 V and 2.5 A. The limiting error (in \%) in computing the load resistance is (up to two decimal places) _____.

**Key:** (2.5)

**Exp:** → The error in voltmeter is

\[(0 - 300V), \pm 1\% \text{ of F.S.D} = \pm 300 \times \frac{1}{100} = \pm 3V\]

Similarly, The error in ammeter is

\[(0 - 5A), \pm 0.5\% \text{ of F.S.D} = \pm 5 \times \frac{0.5}{100} = \pm 0.025A\]

→ The reading voltmeter and ammeter are 200V and 2.5 A (given)

\[\Rightarrow R = \frac{200 \pm 3}{2.5 \pm 0.025}\]

\[= \frac{200 \pm \left(\frac{3}{200} \times 100\right)}{2.5 \pm \left(\frac{0.025}{2.5} \times 100\right)} = \frac{200 \pm 1.5\%}{2.5 \pm 1\%} = \frac{200}{2.5} \pm (1.5 + 1)\% = 80 \pm 2.5\%\]

→ So the \% limiting error in the resistance is 2.5.
29. The Fourier transform of a signal \( x(t) \), denoted by \( X(j\omega) \), is shown in the figure.

Let \( y(t) = x(t) + e^{jt} x(t) \). The value of Fourier transform of \( y(t) \) evaluated at the angular frequency \( \omega = 0.5 \text{ rad/s} \) is 

(A) 0.5 
(B) 1 
(C) 1.5 
(D) 2.5 

**Key:** (C)

**Exp:** Given,

\[
y(t) = x(t) + e^{jt} x(t)
\]

Apply frequency shifting property

\[
Y(j\omega) = X(j\omega) + X(j(\omega - 1))
\]

\[
\Rightarrow y(j0.5) = x(j0.5) + x(j(-0.5))
\]

\[
= 1 + 0.5 = 1.5
\]

30. In the given relaxation oscillator, the opamps and the zener diodes are ideal. The frequency (in kHz) of the square wave output \( v_0 \) is ____.
Key: (200)

Exp:

From above circuit General Scheme of triangular and square forms

\[ V_{th} - V_{nl} = \frac{L_+}{T_1} \]

\[ T_1 = RC \cdot \frac{V_{th} - V_{nl}}{L_+} \]

\[ V_{th} = L_+ \cdot \frac{R_1}{R_1 + R_2} \]

\[ = \left( V_i + V_z \right) \cdot \frac{R_1}{R_1 + R_2} \]

\[ = \frac{5}{2} - 2.5 \]

\[ V_{nl} = +L_+ \cdot \frac{R_1}{R_1 + R_2} = -5 \cdot \frac{R_1}{R_1 + R_2} = -2.5 \]

\[ T_1 = 2.5 \times 10^{-6} \cdot \frac{5}{5} = 2.5 \times 10^{-6} \]

\[ T_2 = RC \cdot \frac{V_{th} - V_{nl}}{-L_-} = 2.5 \times 10^{-6} \]

\[ f = \frac{1}{T_1 + T_2} = 200 \text{ kHz} \]
31. Consider the following system of linear equations:

\[
\begin{align*}
3x + 2ky &= -2 \\
kx + 6y &= 2
\end{align*}
\]

Here \(x\) and \(y\) are the unknowns and \(k\) is real constant. The value of \(k\) for which there are infinite number of solutions is

(A) 3  
(B) 1  
(C) -3  
(D) -6

**Key:** (C)

**Exp:** Consider the system of equations:

\[
\begin{align*}
a_1x + b_1y + c_1 &= 0 \quad \& \quad a_2x + b_2y + c_2 &= 0
\end{align*}
\]

If \(\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}\); then equations has an infinite no. of solutions

Given equations

\[
\begin{align*}
3x + 2ky + 2 &= 0 \\
kx + 6y - 2 &= 0
\end{align*}
\]

\[
\begin{align*}
\therefore \frac{3}{k} = \frac{2k}{6} = \frac{2}{-2}
\end{align*}
\]

\[
\begin{align*}
\Rightarrow \frac{2k}{6} &= \frac{2}{-2} \quad \text{ (or) } \frac{3}{k} = \frac{2}{-2}
\end{align*}
\]

\[
\begin{align*}
\Rightarrow \frac{2k}{6} &= -1 \quad \Rightarrow \frac{3}{k} = -1
\end{align*}
\]

\[
\begin{align*}
\Rightarrow k = -6/2 &= -3 \quad \Rightarrow k = -3
\end{align*}
\]

\(\therefore\) The value of \(k\) is -3 for which the system of eq’s has an infinite no. of solutions.

32. The circuit given uses ideal opamps. The current \(I\) (in \(\mu\)A) drawn from the source \(v_s\) is (up to two decimal places) ____.
Key: (0.99)

Exp:

\[ I = \frac{1}{10k} + \frac{1 - V_0}{10.1k} \]  \quad \text{...(1)}

From above circuit it is cascaded of Inverting amplifier

\[ V_0 = \left( -1 \cdot \frac{20}{10} \right) \left( -\frac{20}{20} \right) = +2V \]

\[ I = \frac{1}{10k} - \frac{1}{10.1k} = \frac{1}{1k} \left[ \frac{10.1 - 10}{101} \right] \]

\[ = \frac{0.1}{101} \text{mA} \]

\[ = 9.9 \times 10^{-4} \times 10^{-3} \]

\[ = 0.99 \mu A \]

Answer = 0.99

33. The Boolean function \( F(X,Y) \) realized by the given circuit is

(A) \( \overline{X}Y + XY \)  \quad (B) \( \overline{X}Y + XY \)

(C) \( X + Y \)  \quad (D) \( X \cdot Y \)
Key: (A)

Exp:

By referring the circuit

\[ F = (\overline{x} + y)(x + \overline{y}) \]

\[ = \overline{x} + y + x + \overline{y} \]

\[ = xy + \overline{xy} \]

\[ = x \oplus y \]

It is a very well known standard 2 input XOR gate implementation circuit only by using 2 input Nand gates. (Directly we can select the option, without doing above simplification steps).

34. Assuming ideal opamp, the RMS voltage (in mV) in the output \( V_0 \) only due to the 230 V, 50 Hz interference is (up to one decimal place)____.

Key: (10.836)

Exp:
We have to calculate $V_0$ (RMS): Make D.C sources zero and $C_2, R_1, C_1,$ does not effect [because op-Amp is ideal, apply virtual ground concept]

$V_0 = -50k \times j \omega C_1 V_i$

$|V_0| = 50k \omega C_1 V_i$

$= 50 \times 1000 \times 2\pi \times f \times 3 \times 10^{-12} \times 230$

RMS voltage $= |V_0| = 10.836 \text{ mV}$

35. Let $y[n] = x[n] \ast h[n]$, where $\ast$ denotes convolution and $x[n]$ and $h[n]$ are two discrete time sequence. Given that the $z$-transform of $y[n]$ is $Y(z) = 2 + 3z^{-1} + z^{-2}$, the $z$-transform of $p[n] = x[n] \ast h[n-2]$ is

(A) $2 + 3z + z^{-2}$

(B) $3z + z^{-2}$

(C) $2z^2 + 3z + 1$

(D) $2z^2 + 3z^{-3} + z^{-4}$

Key: (D)

Exp: $y[n] = x[n] \ast h[n]$

Convolution in linear shift invariant operation,

$x[n] \ast h[n-2] = y[n-2] = p[n]$

Apply time shifting property

$P(z) = z^{-2}Y(z)$

$= 2z^{-2} + 3z^{-3} + z^{-4}$

36. A portion of an assembly language program written for an 8-bit microprocessor is given below along with explanations. The code is intended to introduce a software time delay. The processor is driven by a 5 MHz clock. The time delay (in $\mu$s) introduced by the program is _____.

```
MVI B, 64 H ; Move immediate the given byte into register B. Takes 7 clock periods.
LOOP : DCR B ; Decrement register B. Affects Flags. Take 4 lock periods.
JNZ LOOP ; Jump to address with Label LOOP if zero flag is not set. Takes
10 clock periods when jump is performed and 7 clock periods
when jump is not performed.
```

Key: (280.8)

Exp: $\text{(64)}_{10} = \text{(100)}_{10}$

$\Rightarrow \text{MVI B, 64 H} \Rightarrow \text{B} = \text{(64)}_H = \text{(100)}_{10}$

It will be executed only once $\Rightarrow 7$ clocks
Since B = 100, this loop will run due to true condition (Z = 0), 99 times.

So the number of clock = 99(4 + 10) = 1386

- On the 100th iteration, when B = 0, “JNZ Loop” will take 7 clocks, as the condition is false (Z = 1).

So this 100th iteration needs = 1 (4 + 7) = 11 clocks

- So total

7 + 1386 + 11 = 1404 clocks are needed

- \( f_{\text{clk}} = 5 \text{ MHz} \)

\( T_{\text{clk}} = \frac{1}{5 \times 10^6} = 0.2 \mu\text{sec} \)

1404 clocks = 1404 × 0.2 \( \mu\) sec = 280.8 \( \mu\) sec

37. In the given circuit, superposition is applied. When \( V_2 \) is set to 0V, the current \( I_2 \) is \(-6A\). When \( V_1 \) is set to 0 V, the current \( I_1 \) is +6A. Current \( I_3 \) (in A) when both sources are applied will be (up to two decimal places) _____.

Key: (1)

Exp: Case-1: \([V_2 = 0V, I_2 = -6A]\)
\[ V_{2\Omega} = V_{6\Omega} \text{ (\because 2\Omega \parallel 6\Omega)} \]
\[ I'_1 = \frac{V_{6\Omega}}{6} = \frac{-12}{6} = -2A \]

\[ \rightarrow \text{Case - 2: } [V_i = 0, I_i = 6A] \]

\[ V_{6\Omega} = V_{3\Omega} \]
\[ I''_3 = \frac{V_{6\Omega}}{6} = \frac{18}{6} = 3A \]

\[ \rightarrow \text{By superposition, by referring above 2 circuits we can say} \]
\[ I_3 = I'_3 + I''_3 \]
\[ = (-2) + (3) = 1A \]

38. A 2-bit synchronous counter using two J-K flip flops is shown. The expressions for the inputs to the J-K flip flops are also shown in the figure. The output sequence of the counter starting from \( Q_1Q_2 = 00 \) is

(A) 00 \( \rightarrow \) 11 \( \rightarrow \) 10 \( \rightarrow \) 01 \( \rightarrow \) 00...
(B) 00 \( \rightarrow \) 01 \( \rightarrow \) 10 \( \rightarrow \) 11 \( \rightarrow \) 00...
(C) 00 \( \rightarrow \) 01 \( \rightarrow \) 11 \( \rightarrow \) 10 \( \rightarrow \) 00...
(D) 00 \( \rightarrow \) 10 \( \rightarrow \) 11 \( \rightarrow \) 01 \( \rightarrow \) 00...
**Key:** (C)

**Exp:**

<table>
<thead>
<tr>
<th>Present state</th>
<th>Flipflop</th>
<th>Next State</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q_1$</td>
<td>$Q_2$</td>
<td>$J_1\begin{pmatrix} Q_1+Q_2 \end{pmatrix}$</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

→ By using $J_1,k_1,Q_1$ we get $Q_1^+$

$J_2,k_2,Q_2$ we get $Q_2^+$ with help of state table of JK flip flop

→ By observing the table we can say, the counting pattern of the counter is

$00 \rightarrow 01 \rightarrow 11 \rightarrow 10 \rightarrow 00 \ ...$

39. A high Q coil having districted (self) capacitance is tested with a Q-meter. First resonance at $\omega_1 = 10^6 \text{rad/s}$ is obtained with a capacitance of 990 pF. The second resonance at $\omega_2 = 2 \times 10^6 \text{rad/s}$ is obtained with a 240 pF capacitance. The value of the inductance (in mH) of the coil is (up to one decimal place) ____.

**Key:** (1)

**Exp:** → It is given that

$\omega_1 = 10^6 \text{rad/sec}; \quad C_1 = 990\text{pF}$

$\omega_2 = 2 \times 10^6 \text{rad/sec}; \quad C_2 = 240\text{pF}$

$n = \omega_2 / \omega_1 = 2$

→ In the Q-meter the distributed capacitance is given by

$C_d = \frac{C_1 - n^2C_2}{n^2 - 1}$

$= \frac{990 - 2^2(240)}{2^2 - 1} = 10\text{pF}$

→ We can use

$\omega_1 = \frac{1}{\sqrt{L(C_d + C_1)}} \text{ or } \omega_2 = \frac{1}{\sqrt{L(C_d + C_2)}}$
Any one of these 2 can be used

\[ \Rightarrow \omega_0^2 = \frac{1}{L(C_d + C_i)} \]

\[ \Rightarrow L = \frac{1}{\omega_0^2(C_d + C_i)} \]

\[ = \frac{1}{(10^6)^2(10 + 990)} = 1 \text{ mH} \]

40. Consider a standard negative feedback configuration with \( G(s) = \frac{1}{(s+1)(s+2)} \) and \( H(s) = \frac{s + \alpha}{s} \).

For the closed loop system to have a poles on the imaginary axis, the value of \( \alpha \) should be equal to (up to one decimal place) ____.

**Key:** (9)

**Exp:** Given \( G(s) = \frac{1}{(s+1)(s+2)} \)

\[ H(s) = \frac{s + \alpha}{s} \]

\[ \text{C.E} = 1 + G(s)H(s) = 0; \]

\[ s(s+1)(s+2) + (s + \alpha) = 0 \]

\[ s^3 + 3s^2 + 2s + s + \alpha = 0 \]

\[ s^3 + 3s^2 + 3s + \alpha = 0 \]

If system is marginal stable

\[ 3 \times 3 = \alpha \]

\[ \alpha = 9 \]

41. The inductance of a coil is measured using the bridge shown in the figure. Balance (\( D = 0 \)) is obtained with \( C_1 = 1 \text{ nF} \). \( R_1 = 2.2 \text{ M}\Omega \), \( R_2 = 22.2 \text{ k}\Omega \), \( R_3 = 10 \text{ k}\Omega \). The value of the inductance \( L_x \) (in mH) is ____.
Key: (222)

Exp: \(\rightarrow\) From the given circuit

\[
Z_i = R_1 \parallel \frac{1}{j\omega C_1} = \frac{R_i}{j\omega C_1} = \frac{R_1}{1 + j\omega C_1 R_i}
\]

\[
Z_x = R_2
\]

\[
Z_3 = R_x + j\omega L_x
\]

\[
Z_4 = R_4
\]

\(\rightarrow\) For Bridge balance

\[
Z_i Z_3 = Z_2 Z_4
\]

\[
\Rightarrow \left[ \frac{R_1}{1 + j\omega C_1 R_i} \right] \left[ R_x + j\omega L_x \right] = R_2 R_4
\]

\[
\Rightarrow R_1 \left[ R_x + j\omega L_x \right] = R_2 R_4 \left[ 1 + j\omega C_1 R_i \right]
\]

\[
\Rightarrow R_1 R_x + j\omega L_x R_1 = R_2 R_4 + j\omega C_1 R_4 R_2 R_4
\]

Equating the imaginary part we have

\[
\omega L_x R_1 = \omega C_1 R_4 R_2 R_4
\]

\[
\Rightarrow L_x = \frac{C_1 R_4 R_2}{R_1}
\]

\[
= 1 \times 10^{-3} \times 2.2 \times 10^6 \times 10 \times 10^3
\]

\[
= 222 \text{ mH}
\]

42. A multi-mode optical fiber with a large core diameter has a core refractive index \(n_1 = 1.5\) and cladding refractive index \(n_2 = 1.4142\).

The maximum value of \(\theta_\lambda\) (in degrees) for which the incident light from air will be guided in the optical fiber is \(\pm \) _______.

\[
\text{Light} \quad \text{Cladding } n_2 = 1.4142
\]

\[
\theta_\lambda \quad \text{Core } n_1 = 1.5
\]

\[
\text{Air} \quad \text{Cladding } n_2 = 1.4142
\]
Key: (30)

Exp: \(\rightarrow\) It is given that 

Refractive index of core \([n_1 = 1.5]\)

Refractive index of cladding \([n_2 = 1.4142]\)

\(\rightarrow\) We know

\[\sin \theta_A = \sqrt{n_1^2 - n_2^2}\] (Where \(\theta_A\) is acceptance angle)

\[\Rightarrow \theta_A = \sin^{-1}\sqrt{n_1^2 - n_2^2}\]

\[= \sin^{-1}\left[\sqrt{(1.5)^2 - (1.4142)^2}\right]\]

\[= 30^\circ\]

43. The product of sum expression of a Boolean function \(F(A, B, C)\) of three variables is given by

\[F(A, B, C) = (A + B + \bar{C})(A + \bar{B} + \bar{C})(A + B + C)(\bar{A} + \bar{B} + \bar{C})\]

The canonical sum of product expression of \(F(A, B, C)\) is given by

(A) \(\bar{A}BC + \bar{A}BC + \bar{A}BC + \bar{A}BC\)

(B) \(\bar{A}BC + \bar{A}BC + \bar{A}BC + \bar{A}BC\)

(C) \(ABC + \bar{A}BC + \bar{A}BC + \bar{A}BC\)

(D) \(\bar{A}BC + \bar{A}BC + \bar{A}BC + \bar{A}BC\)

Key: (B)

Exp: \(F(A, B, C) = (A + B + \bar{C})(A + \bar{B} + \bar{C})(A + B + C)(\bar{A} + \bar{B} + \bar{C})\)

\[= \Pi m(1, 3, 4, 7)\]

\[= \Sigma M(0, 2, 5, 6)\]

\[= m_0 + m_2 + m_3 + m_6\]

\[= \bar{A}BC + \bar{A}BC + \bar{A}BC + \bar{A}BC\]

44. Unit step resonance of a linear time invariant (LT) system is given by \(y(t) = \left(1 - e^{-2t}\right)u(t)\).

Assuming zero initial condition, the transfer function of the system is

(A) \(\frac{1}{s + 1}\)  (B) \(\frac{2}{(s + 1)(s + 2)}\)  (C) \(\frac{1}{s + 2}\)  (D) \(\frac{2}{s + 2}\)

Key: (D)

Exp: Given unit step response
\[ y(t) = (1 - e^{-2t})u(t) \]
impulse response \( h(t) = \frac{d}{dt} y(t) \)
\[ h(t) = \delta(t) - e^{-2t}\delta(t) + 2e^{-2t}u(t) \]
\[ = 2e^{-2t}u(t) \]

\[ L\{h(t)\} = \text{Transfer function} \]
\[ T.F = L\{2e^{-2t}4(t)\} = \frac{2}{s + 2} \]
Option ‘D’

45. A Michelson Interferometer using a laser source of wavelength \( \lambda_0 = 500 \text{ nm} \), with both the mirrors (M₁ & M₂) fixed and positioned equidistant from the splitter/combiner is shown in the figure. When a dielectric plates of refractive index \( n = 1.5 \), of thickness \( t \), is placed in front of the mirror M₂, a dark fringe is observed on the detector. When the dielectric plate is removed without changing the position of the mirrors M₁ & M₂, a bright fringe is observed on the detector. The minimum thickness \( t \) (in nm) of the dielectric is _____.

**Key:** (250)

**Exp:** Path difference \( \frac{\lambda_0}{2} \)

\[ \text{Path difference} = 2(N - 1)t \]
\[ \Rightarrow 2(N - 1)t = \frac{\lambda_0}{2} \]
\[ \Rightarrow 2(1.5 - 1)t = \frac{500\text{nm}}{2} \]
\[ \Rightarrow t = 250\text{nm} \]
46. An opamp that is powered from a ±5V supply is used to build a non-inverting amplifier having a gain of 15. The slew rate of the opamp is $0.5 \times 10^6$ V/s. For a sinusoidal input with amplitude of 0.3 V, the maximum frequency (in kHz) up to which it can be operated without any distortion is (up to one decimal place) _____.

**Key:** (17.68)

**Exp:** Slew Rate (SR) = \( \frac{\Delta V_o}{\Delta t} = \left( \frac{V}{\mu \text{sec}} \right) \)

\[ V_o = A V_i \sin \omega_m t \]
\[ = 15 \times 0.3 \sin \omega_m t \]
\[ \left| \frac{dv_o}{dt} \right|_{\text{max}} = 4.5\omega_m \cos \omega t = 4.5\omega_m \]
\[ 0.5 \times 10^6 = 4.5 \times 2\pi \times f_{\text{max}} \]
\[ f_{\text{max}} = 17.68 \text{ kHz} \]

47. The average velocity \( v \) of flow clear water in a 100 cm (inner) diameter tube is measured using the ultrasonic flow meter as shown in the figure. The angle \( \theta \) is 45°. The measured transit times are \( t_1 = 0.9950 \) ms and \( t_1 = 1.0000 \) ms. The velocity \( v \) (in m/s) in the pipe is (up to one decimal place) _____.

**Key:** (0.005025)

**Exp:**

By referring the above diagram, we have

\[ \sin 45^\circ = \frac{100}{d} \]
\[ \Rightarrow d = 100\sqrt{2} \]
→ It is given that
\[ t_1 = 0.9950 \text{ m.sec} \]
\[ \Rightarrow f_1 = \frac{1}{t_1} = \frac{1}{0.9950 \times 10^{-3}} = 1.005025 \times 10^3 \text{ Hz} \]

Similarly,
\[ t_2 = 1.0000 \text{ m.sec} \]
\[ f_2 = f_1 - f_2 = 0.005025 \]
\[ \Delta f = \frac{2V_i \cos \theta}{t} \] (well known standard formula)
\[ \Rightarrow V_i = \frac{(\Delta f)d}{2 \cos \theta} = \frac{0.005025 \times 100 \sqrt{2}}{2 \times \cos 45^\circ} = 0.5025 \text{ cm/sec} = 0.005025 \text{ m/sec} \]

48. Given \( \bar{F} = (x^2 - 2y)\hat{i} - 4yz\hat{j} + 4xz^2\hat{k} \), the value of the line integral \( \int_C \bar{F} \cdot d\bar{r} \) along the straight line \( c \) from \((0,0,0)\) to \((1,1,1)\) is

(A) \( \frac{3}{16} \)  \hspace{1cm} (B) 0  \hspace{1cm} (C) \( -\frac{5}{12} \)  \hspace{1cm} (D) \( -1 \)

**Key:** (D)

**Exp:** Given,
\[ \bar{F} = (x^2 - 2y)\hat{i} - 4yz\hat{j} + 4xz^2\hat{k} \]

.: The equations of the straight line from \((0,0,0)\) to \((1,1,1)\) is
\[ \frac{x - 0}{1 - 0} = \frac{y - 0}{1 - 0} = \frac{z - 0}{1 - 0} = t \]
\[ \Rightarrow x = y = z = t \]
\[ \Rightarrow dx = dy = dz = dt. \]
If \((x, y, z) = (0, 0, 0)\) then \( t = 0 \)
If \((x, y, z) = (1, 1, 1)\) then \( t = 1 \)

.: The line integral
\[ \int_C \bar{F} \cdot d\bar{r} = \int_{C(0,0)}^{(1,1)} [(x^2 - 2y)\hat{i} - 4yz\hat{j} + 4xz^2\hat{k}] \cdot [dx\hat{i} + dy\hat{j} + dz\hat{k}] \]
where \( d\bar{r} = dx\hat{i} + dy\hat{j} + dz\hat{k} \)
\[ = \int_{(0,0)}^{(1,1)} (x^2 - 2y)dx - 4yzdy + 4xz^2dz \]
\[
= \int_{t=0}^{1} \left( t^2 - 2t \right) dt - 4t^2 dt + 4t^3 dt \quad \left[ \because x = y = z = t ; dx = dy = dz = dt \right]
\]

\[
= \int_{t=0}^{1} \left[ t^2 - 2t - 4t^2 + 4t^3 \right] dt
\]

\[
= \int_{t=0}^{1} \left[ 4t^3 - 3t^2 - 2t \right] dt
\]

\[
= 4 \left( \frac{t^4}{4} \right) - 3 \left( \frac{t^3}{3} \right) - 2 \left( \frac{t^2}{2} \right) \bigg|_{t=0}^{t=1}
\]

\[
= (1 - 1 - 1) - (0 - 0 - 0) = -1.
\]

\[
\Rightarrow \int_{C} F \cdot d\vec{l} = -1.
\]

49. Two bags A and B have equal number of balls. Bag A has 20% red balls and 80% green balls. Bag B has 30% red balls, 60% green balls and 10% yellow balls. Contents of Bags A and B are mixed thoroughly and a ball is randomly picked from the mixture. What is the chance that the ball picked is red?

(A) 20%  
(B) 25%  
(C) 30%  
(D) 40%

**Key:** (B)

**Exp:** Given that, two bags A and B have equal no. of balls

Let us assume that,

No. of balls in bag A = No. balls in bag B = 100.

\[
\therefore \text{Total no. of balls} = 100 + 100 = 200.
\]

Also given that, Bag A has 20% red balls and Bag B has 30% red balls.

\[
\therefore \text{Total no. of red balls} = 20 + 30 = 50.
\]

\[
\therefore \text{Required probability} = \frac{50C_1}{200C_1} = \frac{50}{200} = \frac{1}{4} = 25\%.
\]

50. For the sequence \( x[n] = \{1,-1,1,-1\} \), with \( n = 0,1,2,3 \), the DFT is computed as \( X(k) = \sum_{n=0}^{3} x[n] e^{-j\frac{2\pi nk}{4}} \), for \( k = 0,1,2,3 \). The value of \( k \) for which \( X(k) \) is not zero is

(A) 0  
(B) 1  
(C) 2  
(D) 3

**Key:** (C)

**Exp:** For 4-point DFT,
\[
\begin{bmatrix}
X(0) \\
X(1) \\
X(2) \\
X(3)
\end{bmatrix} =
\begin{bmatrix}
1 & 1 & 1 & 1 \\
1 & -j & 1 & j \\
1 & -1 & 1 & -1 \\
1 & j & 1 & -j
\end{bmatrix}
\begin{bmatrix}
x(0) \\
x(1) \\
x(2) \\
x(3)
\end{bmatrix}
\]

For given \( x(n) \{1, 1, 1, 1\} \)

\[
\begin{bmatrix}
X(0) \\
X(1) \\
X(2) \\
X(3)
\end{bmatrix} =
\begin{bmatrix}
1 & 1 & 1 & 1 \\
1 & -j & 1 & j \\
1 & -1 & 1 & -1 \\
1 & j & 1 & -j
\end{bmatrix}
\begin{bmatrix}
0 \\
0 \\
4 \\
0
\end{bmatrix}
\]

Thus \( x(2) \) in only non-zero \( \Rightarrow k = 2 \)

51. Consider the linear system
\[
\dot{x} = \begin{bmatrix}
-1 & 0 \\
0 & -2
\end{bmatrix} x,
\]
with initial condition \( x(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \). The solution \( x(t) \) for this system is

(A) \( x(t) = \begin{bmatrix} e^{-t} & te^{-2t} \\ 0 & e^{-2t} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \)

(B) \( x(t) = \begin{bmatrix} e^{-t} & 0 \\ 0 & e^{2t} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \)

(C) \( x(t) = \begin{bmatrix} e^{-t} & -t^2e^{-2t} \\ 0 & e^{-2t} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \)

(D) \( x(t) = \begin{bmatrix} e^{-t} & 0 \\ 0 & e^{-2t} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \)

Key: (D)

Exp: Given
\[
\dot{x} = \begin{bmatrix}
-1 & 0 \\
0 & -2
\end{bmatrix} x \quad \therefore -A = \begin{bmatrix}
-1 & 0 \\
0 & -2
\end{bmatrix}
\]

\[
x(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}
\]

By state equations
\[
x(t) = e^{At} x(0)
\]

\( e^{At} = \) State transition matrix \( L^{-1} [sI - A]^{-1} \)

\[
sI - A = \begin{bmatrix}
 s + 1 & 0 \\
0 & s + 2
\end{bmatrix}
\]

\[
[sI - A]^{-1} = \frac{1}{(s + 1)(s + 1)} \begin{bmatrix}
 s + 2 & 0 \\
0 & s + 1
\end{bmatrix}
\]
\[
\begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
s + 1 \\
s + 2
\end{bmatrix}
\]

\[e^{At} = \phi(t) = L^{-1}\left\{(sI - A)^{-1}\right\}
\]

\[
= \begin{bmatrix}
\frac{1}{s+1} & 0 \\
0 & \frac{1}{s+2}
\end{bmatrix}
\]

\[x(t) = e^{At}x(0) = \begin{bmatrix}
e^{-t} & 0 \\
0 & e^{-2t}
\end{bmatrix}\begin{bmatrix}1 \\
1\end{bmatrix}
\]

52. The sampling rate for Compact Discs (CDs) is 44,000 samples/s. If the samples are quantized to 256 levels and binary coded, the corresponding bit rate (in bits per second) is ____.

**Key:** (352000)

**Exp:** Given,

Sampling Rate = 44,000 Samples / sec

Number of Quantization levels = 256.

No. of Bits required for each sample = Log to the base 2 (256) = 8 bits/sample.

This we got under the assumption that all levels are equiprobable.

Thus bit rate = 8 bits/sample X 44,000 Samples/sec = 3,52,000 bits/sec

53. In the figure, an RLC load is supplied by a 230 V, 50 Hz single phase source. The magnitude of the reactive power (in VAr) supplied by the source is ____.

**Key:** (67.36)

**Exp:** \[
\rightarrow \hat{I} = \frac{230[0^\circ]}{162.6 + j162.6} = \frac{230[0^\circ]}{\sqrt{162.6^2 + 162.6^2}} \tan^{-1} 1 = \frac{230[0^\circ]}{230[45^\circ]} = l[45^\circ]
\]
\[
\mathbf{I}_2 = \frac{230[0^\circ]}{230[-90^\circ]} = 1[90^\circ]
\]
\[
\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2
\]
\[
= [1[-45^\circ]] + [1[90^\circ]]
\]
\[
= 1[\cos(-45^\circ) + j\sin(-45^\circ)] + 1[\cos90^\circ + j\sin90^\circ]
\]
\[
= \frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}} + j = \frac{1}{\sqrt{2}} + j\left(1 - \frac{1}{\sqrt{2}}\right)
\]
\[
= \frac{1}{\sqrt{2}} + j0.293
\]
\[
= 0.707 + j0.293 = 0.765[22.51]
\]
\[
\rightarrow \text{All the given information's are in the R.M.S form by default}
\]
\[
Q_{\text{source}} = |\bar{V}_{\text{source}}| |\bar{I}_{\text{source}}| \sin[\theta_v - 0_j]
\]
\[
= 230 \times 0.765 \times \sin(0 - 22.51) \quad [\because \text{Assuming voltage as reference phaser } \theta_v = 0]
\]
\[
= 67.36 \text{ VAR}
\]

54. Consider the following equations
\[
\frac{\partial V(x, y)}{\partial x} = px^2 + y^2 + 2xy
\]
\[
\frac{\partial V(x, y)}{\partial y} = x^2qy^2 + 2xy
\]

Where \(p\) and \(q\) are constants. \(V(x, y)\) that satisfies the above equations is

\[
(A) \quad p\frac{x^3}{3} + q\frac{y^3}{3} + 2xy + 6
\]
\[
(B) \quad p\frac{x^3}{3} + q\frac{y^3}{3} + 5
\]
\[
(C) \quad p\frac{x^3}{3} + q\frac{y^3}{3} + x^2y + xy^2 + xy
\]
\[
(D) \quad p\frac{x^3}{3} + q\frac{y^3}{3} + x^2y + xy^2
\]

**Key:** (D)

**Exp:** By total differential of \(V\), we have
\[
dV = \frac{\partial V}{\partial x}dx + \frac{\partial V}{\partial y}dy
\]
\[
\Rightarrow dV = (px^2 + y^2 + 2xy)dx + (x^2 + qy^2 + 2xy)dy;
\]

Since \(\frac{\partial V}{\partial x} = px^2 + y^2 + 2xy\) and \(\frac{\partial V}{\partial y} = x^2 + qy^2 + 2xy\).
Le \( M = px^2 + y^2 + 2xy \) & \( N = x^2 + qy^2 + 2xy \)

\[ \Rightarrow \frac{\partial M}{\partial y} = 2y + 2x ; \quad \Rightarrow \frac{\partial N}{\partial x} = 2x + 2y \]

\[ \therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \]

\[ \therefore \text{Eq}(1) \text{is exact.} \]

\[ \therefore \text{The solution of eq}(1) \]

\[ \int dv = \int (px^2 + y^2 + 2xy) \, dx + \int (x^2 + qy^2 + 2xy) \, dy \]

[\text{Treat 'y' terms as constants}][\text{Integrate w.r.t 'y' only for 'y' terms}]

\[ \Rightarrow v(x, y) = p \left( \frac{x^3}{3} \right) + y^2(x) + 2y(x^2/2) + q(y^3/3). \]

\[ \Rightarrow v(x, y) = p \frac{x^3}{3} + q \frac{y^3}{3} + x^2y + xy^2. \]

55. Consider the standard negative feedback configuration with \( G(s) = \frac{s^2 + 0.2s + 100}{s^2 - 0.2s + 100} \) and \( H(s) = \frac{1}{2} \).

The number of clockwise encirclements of \((-1, 0)\) in the Nyquist plot of the Loop transfer-function \( G(s)H(s) \) is_____.

**Key:** (0)

**Exp:**

\( G(s) = \frac{s^2 + 0.2s + 100}{s^2 - 0.2s + 100} ; \quad H(s) = \frac{1}{2} \)

Loop function = \( G(s)H(s) \)

\[ = \frac{1}{2} \frac{s^2 + 0.2s + 100}{s^2 - 0.2s + 100} \]

\[ = \frac{1}{2} \angle 2 \arctan \left( \frac{0.2}{100 - \omega^2} \right) \angle -180^\circ \]

\[ G(j\omega + j\omega); \quad \omega = 0; \quad M = \frac{1}{2} \angle -180^\circ \]

\[ \omega = \infty; \quad M = \frac{1}{2} \angle 0^\circ \]
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