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GATE 2018
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COMPUTER SCIENCE ENGINEERING

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Section-I: General Ability

1. What would be the smallest natural number which when divided either by 20 or by 42 or by 76 leaves a remainder of 7 in each case?
   (A) 3047   (B) 6047   (C) 7987   (D) 63847

   **Key:** (C)

   **Sol:** The smallest number when divided by 20, 42, 76 and leaves a remainder ‘7’ in each case
   
   \[ \text{LCM} (20, 42, 76) + 7 = 7980 + 7 = 7987 \]

2. The area of a square is d. What is the area of the circle which has the diagonal of the square as its diameter?
   (A) \( \pi d \)   (B) \( \pi d^2 \)   (C) \( \frac{1}{4} \pi d^2 \)   (D) \( \frac{1}{2} \pi d \)

   **Key:** (D)

   **Sol:** Given, Area of square is d.
   
   Let us assume that the side of square as ‘a’
   \[ a^2 = d \Rightarrow d = a^2 \]
   
   Given that diameter of circle = diagonal of the square = \( \sqrt{2} a \)
   \[ \therefore \text{Radius of circle} = \frac{\sqrt{2} a}{2} = \frac{a}{\sqrt{2}} \]
   \[ \therefore \text{The area of circle} = \pi \left( \frac{a}{\sqrt{2}} \right)^2 = \frac{\pi a^2}{2} = \frac{\pi}{2} d \]
   \[ \therefore a^2 = d \]
   
   So option is (D).

3. "A ________ investigation can sometimes yield new facts, but typically organized ones are more successful"

   The word that best fill the blanks in the above sentence is
   (A) meandering   (B) timely   (C) consistent   (D) systematic

   **Key:** (A)

4. "From where are they bringing their books? ________ bringing ________ books from ______"

   The words that best fill the blanks in the above sentence are
   (A) Their, they’re, there   (B) They’re, their, there
   (C) There, their, they’re   (D) They’re, there, there

   **Key:** (B)

5. What is the missing number in the following sequence?

   2, 12, 60, 240, 720, 1440, _____, 0

   (A) 2880   (B) 1440   (C) 720   (D) 0

   \[ \text{Given sequence} = 2, 12, 60, 240, 720, 1440, 2880, 0 \]
   \[ \therefore \text{Missing number} = 2880 \]
6. In a party, 60% of the invited guests are male and 40% are female. If 80% of the invited guests attended the party and if all the invited female guests attended, what would be the ratio of males to females among the attendees in the party?

(A) 2:3  (B) 1:1  (C) 3:2  (D) 2:1

Key: (B)

Sol: Given that, No. of invited male guests = 60
No. of invited female guests = 40
Total no. of invited people = 100 [60+40],

Given, Out of 100; all 40 female guests are attended.

Total no.of attended guests = 80.

∴ No.of attended male guests = 80-40 = 40
∴ The required ratio = 40 : 40 = 1:1.

7. In the figure below, \( \angle DEC + \angle BFC \) is equal to \( \_\_\_\_\_\_\_\_ \). 

(A) \( \angle BCD - \angle BAD \)  (B) \( \angle BAD + \angle BCF \) 
(C) \( \angle BAD + \angle BCD \)  (D) \( \angle CBA + \angle ADC \)

Key: (A)

Sol: \( \text{In } \Delta AEB; \angle A + \angle E + \angle B = 180^\circ \) \hspace{1cm} ... (1)
\( \text{In } \Delta FDA; \angle F + \angle D + \angle A = 180^\circ \) \hspace{1cm} ... (2)

From (1) and (2) \( \rightarrow \angle A + \angle B + \angle D + \angle E + \angle F = 360^\circ \)
\( \angle A + \angle B + \angle D = 360^\circ - (\angle A + \angle E + \angle F) \) \hspace{1cm} ... (3)

\( \text{In Quadrilateral } ABCD; \angle A + \angle B + \angle C + \angle D = 360^\circ \) \hspace{1cm} ... (4)

\( \Rightarrow 360^\circ - (\angle A + \angle E + \angle F) = 360^\circ - \angle C \)
\( \Rightarrow \angle A + \angle E + \angle F = \angle C \)
\( \Rightarrow \angle E + \angle F + \angle C - \angle A \)
\( \Rightarrow \angle DEC + \angle BFC = \angle BCD - \angle BAD \)

8. A six sided unbiased die with four green faces and two red faces is rolled seven times. Which of the following combinations is the most likely outcome of the experiment?

(A) Three green faces and four red faces  (B) Four green faces and three red faces
(C) Five green faces and two red faces  (D) Six green faces and one red face

Key: (B)

Sol: \( \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 \times 0 \)

\( 2 \times 12 \times 60 \times 240 \times 720 \times 1440 \times 1440 \times 0 \)

So option is B
Key: (C)
Sol:  Given that a die with 4-Green; 2-Red.

\[ P(\text{Green faces}) = \frac{4}{6} = \frac{2}{3} = 0.67 \]

\[ P(\text{Red faces}) = \frac{2}{6} = \frac{1}{3} = 0.33 \]

\[ \Rightarrow \text{Best combination } \rightarrow 5 \text{ Green} + 2 \text{ Red} \]

(: Probability of getting green faces is more than double the probability of getting Red faces)

9. In appreciation of the social improvements completed in a town, a wealthy philanthropist decided to
gift Rs. 750 to each male senior citizen in the town and Rs. 1000 to each female senior citizen.
Altogether, there were 300 senior citizens eligible for this gift. However, only 8/9th of the eligible men
and 2/3rd of the eligible women claimed the gift. How much money (in Rupees) did the
philanthropist give away in total?

(A) 1,50,000  (B) 2,00,000  (C) 1,75,000  (D) 1,51,000

Key: (B)
Sol:  Let us assume that No. of men = x (senior citizen)

\[ \therefore \text{ no. of women} = 300 - x \text{ (senior citizen)} (\because \text{ total no. of senior citizen} = 300) \]

\[ \therefore \text{ The amount of money need to paid} \]

\[ = \frac{8x}{9} \times 750 + \frac{2}{3}(300 - x)\times1000 \]

\[ = \left( \frac{8x}{3} \right) \times 250 + \left( 200 - \frac{2x}{3} \right) \times 1000 \]

\[ = \frac{2000x}{3} + 200000 - \frac{2000x}{3} = 2,00,000 \]

10. If \( pqr \neq 0 \) and \( p^x = \frac{1}{q}, q^y = \frac{1}{r}, r^z = \frac{1}{q} \), what is the value of the product \( xyz \)?

(A) \(-1\)  (B) \(\frac{1}{pqr}\)  (C) 1  (D) \(pqr\)

Key: (C)
Sol:  \( p^{-x} = \frac{1}{q} \)

\[ \Rightarrow \left( r^z \right)^x = \frac{1}{q} \]

\[ \Rightarrow r^{zx} = \frac{1}{p} \]

\[ \Rightarrow p = r^z \]

\[ \Rightarrow r^{zx} = \frac{1}{q} \]

\[ \Rightarrow \left( q^y \right)^z = \frac{1}{r} \]

\[ \Rightarrow q^{yz} = q^1 \]

\[ \Rightarrow xyz = 1 \]
1. Let $\oplus$ and $\odot$ denote the Exclusive OR and Exclusive NOR operations, respectively. Which of the following is NOT CORRECT?

(A) $P \oplus Q = P \odot Q$

(B) $\overline{P} \oplus Q = P \odot Q$

(C) $\overline{P} \oplus \overline{Q} = P \oplus Q$

(D) $(P \oplus \overline{P}) \oplus Q = (P \odot \overline{P}) \odot \overline{Q}$

**Key:** (D)

$\implies \overline{P} \oplus \overline{Q} = P \odot Q$ (well known true relation)

$\implies \overline{P} \oplus Q = (P)Q + \overline{P} \overline{Q} = PQ + \overline{P} \overline{Q} = P \odot Q (True)$

$\implies \overline{P} \oplus \overline{Q} = (P)\overline{Q} + (P)(\overline{Q}) = PQ + \overline{P} \overline{Q} = P \oplus Q (True)$

$\implies (P \oplus \overline{P}) \oplus Q = 1 \oplus Q = \overline{Q}$

$(P \odot \overline{P}) \odot \overline{Q} = Q \odot \overline{Q} = Q$

Exp: $So, (P \oplus \overline{P}) \oplus Q = (P \odot \overline{P}) \odot \overline{Q} (False)$

2. Consider the sequential circuit shown in the figure, where both flip-flops used are positive edge-triggered D flip-flops.

The number of states in the state transition diagram of this circuit that have a transition back to the same state on some value of "in" is _______.

**Key:** (2)

Exp:

Let us obtain the counting pattern of the sequential circuit for $IN=0$ and $IN=1$
If we notice both tables we can say

We can combine above 4 and draw the overall state diagram
So, there are 2 states 00 and 11 where transition is happening to back the same state.

3. Consider the following C program

```c
#include<stdio.h>
struct Ournode{
    char x, y, z;
};
int main(){
    struct Ournode p = {'1', '0', 'a' +2};
    struct Ournode *q = &p;
    printf("%c, %c", *((char*)q+1), *((char*)q+2));
    return 0;
}
```

The output of this program is:
(A) 0, c  (B) 0, a+2  (C) ‘0’, ‘a+2’  (D) ‘0’, ‘c’

Key: (A)

Exp: In structure declaration ‘a’ +2 will be adding 2 to ASCII value of a, which results in character c.
Structure pointer is printing character for second element and third element by typecasting it to character pointer and printing value pointed by them hence result will print character 0 and c.

4. In an Entity-Relationship (ER) model, suppose R is a many-to-one relationship from entity set \( E_1 \) to entity set \( E_2 \). Assume that \( E_1 \) & \( E_2 \) participate totally in \( R \) and that the cardinality of \( E_1 \) is greater than the cardinality of \( E_2 \).
Which one of the following is true about \( R \)?
(A) Every entity in \( E_1 \) is associated with exactly one entity in \( E_2 \)
(B) Some entity in \( E_1 \) is associated with more than one entity in \( E_2 \)
(C) Every entity in \( E_2 \) is associated with exactly one entity in \( E_1 \)
(D) Every entity in \( E_2 \) is associated with at most one entity in \( E_1 \)
5. The following are some events that occur after a device controller issues an interrupt while process \( L \) is under execution.

(P) The processor pushes the process status of \( L \) onto the control stack

(Q) The processor finishes the execution of the current instruction.

(R) The processor executes the interrupt service routine.

(S) The processor pops the process status of \( L \) from the control stack.

(T) The processor loads the new PC value based on the interrupt.

Which one of the following is the correct order in which the events above occur?

(A) QPTRS  

(B) PTRSQ  

(C) TRPQS  

(D) QTPRS

Key: (A)

Exp: Events are in order; first current instruction is completed; it pushes the status in control stack; initialize the program counter; execute the ISR for device controller; and resume the old process. This is described by option (A).

6. Let \( N \) be an NFA with \( n \) states. Let \( k \) be the number of states of a minimal DFA equivalent to \( N \). Which one of the following is necessarily true?

(A) \( k \geq 2^n \)  

(B) \( k \geq n \)  

(C) \( k \leq n^2 \)  

(D) \( k \leq 2^n \)

Key: (D)

Exp: NFA can be converted into minimal DFA by using subset construction method. If NFA is having \( n \) states then subset construction method may results in maximum \( 2^n \) states hence \( k \), which is number of states in minimal DFA should be less than \( 2^n \).

7. Consider the following C program:

```c
#include <stdio.h>

int counter = 0;

int calc(int a, int b){
    int c;
    counter++;
    if (b==3) return (a*a*a);
    else {
        c = calc(a, b/3);
    }
    return c;
}
```
int main(){
    calc (4, 81);
    printf ("%d", counter);
}

The output of this program is______

Key: (4)

EXP: The value printed here is the value of counter.

int calc(4, 81){
    int c;
    counter++;  // Counter =1
    if (b==3)return
        a*a*a);
    else {
        c = calc(3,27);
        return(c*c*c);
    }
    counter++;  // Counter is 2
}

int calc(3,27){
    int c;
    counter++;  // Counter is 3
    if (b==3)return
        (a*a*a);
    else {
        c = calc(3,9);
        return(c*c*c);
    }
    counter++;  // Counter is 4
}

int calc(3,9){
    int c;
    counter++;
}

int calc(3,3){
    int c;
    counter++;  // Counter is 5
    if (b==3)return
        (a*a*a);
    else {
        c = calc(3,3);
        return(c*c*c);
    }
    counter++;  // Counter is 6
}

if (b==3)return
    (a*a*a);
else {
    c = calc(3,3);
    return(c*c*c);
}

The output of this program is______
8. Consider the following processor design characteristics.
   I. Register-to-register arithmetic operations only
   II. Fixed-length instruction format
   III. Hardwired control unit
Which of the characteristics above are used in the design of a RISC processor?
(A) I and II only  (B) II and III only  (C) I and III only  (D) I. II and III

Key:  (B)

Exp: Characteristics of RISC
   i. Fixed length instructions
   ii. Hardwired controller instructions
   iii. Load/store architecture (Also called Register to Register architecture)
   iv. One instruction/ Cycle
   v. It allows successive for pipeline
Register to Register Arithmetic operation is happen in RICS machine but statement I says 'only' that is not correct statement.

9. The postorder traversal of a binary tree is 8, 9, 6, 7, 4, 5, 2, 3, 1. The inorder traversal of the same tree is 8, 6, 9, 4, 7, 2, 5, 1, 3. The height of a tree is the length of the longest path from the root to any leaf. The height of the binary tree above is

Key:  (4)

Exp: Following is the tree results from post order and in order traversal:

```
     1
   /   \
  2     3
 / \
4   5
/ \
6   7
/ \
8   9
```
The height of the above tree is 4

10. Consider a long-lived TCP session with an end-to-end bandwidth of 1 Gbps (=10^9 bits-per-second). The session starts with a sequence number of 1234. The minimum time (in seconds, rounded to the closest integer) before this sequence number can be used again is ___.
Key: (34)
Exp: B = 1 Gbps
  1 sec → 10^9 bits
  1 sec → \( \frac{10^9}{8} \) Bytes
  1 sec → \( \frac{10^9}{8} \) sequence numbers (\( \therefore \) TCP assigns sequence number for each and every byte)
  \( 2^{32} \) → 2^32 sequence numbers
  \( \frac{2^{32} \times 8}{10^9} \) sec → 2^32 sequence numbers
  34.35 sec of wrap around time is needed.

11. Consider a matrix \( A = uv^T \) where \( u = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, v = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \). Note that \( v^T \) denotes the transpose of \( v \). The largest eigen value of \( A \) is ________.
Key: (3)
Exp: \( A = uv^T = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} \)
  Characteristic equation of \( A \) is
  \( \lambda^2 - (\text{trace of } A)\lambda + |A| = 0 \)
  \( \Rightarrow \lambda^2 - 3\lambda + 0 = 0 \)
  \( \Rightarrow \lambda(\lambda - 3) = 0 \)
  \( \Rightarrow \lambda = 0, 3 \)
  \( \Rightarrow \text{Largest eigen value } = 3 \)

12. Consider the following statements regarding the slow start phase of the TCP congestion control algorithm. Note that \( cwnd \) stands for the TCP congestion window and MSS denotes the Maximum Segment Size.
(i) The \( cwnd \) increases by 2 MSS on every successful acknowledgment
(ii) The \( cwnd \) approximately doubles on every successful acknowledgment
(iii) The \( cwnd \) increases by 1 MSS on every round trip time
(iv) The \( cwnd \) approximately doubles every round trip time
Which one of the following is correct?
(A) Only (ii) and (iii) are true
(B) Only (i) and (iii) are true
(B) Only (iv) is true
(D) Only (i) and (iv) are true
Key: (C)

Exp: In slow start phase of TCP congestion control, the congestion window size is exponentially increased, in other words, the congestion window size is doubled for every RTT.

13. Which one of the following statements is FALSE?
(A) Context-free grammar can be used to specify both lexical and syntax rules.
(B) Type checking is done before parsing.
(C) High-level Language programs can be translated to different Intermediate Representations.
(D) Arguments to a function can be passed using the program stack.

Key: (B)

Exp: (A) Since all regular languages are context free we can use context free grammar for specifying lexical and syntax rule hence TRUE.
(B) Type checking is done in semantic analysis phase which comes after parsing hence FALSE.
(C) There are different intermediate representation e.g. postfix expression, syntax tree, 3 address code and high level language can be translated into different representations hence TRUE.
(D) In activation record there is a field called actual parameter used for passing argument for the function hence TRUE.

14. Match the following:

<table>
<thead>
<tr>
<th>Field</th>
<th>Length in bits</th>
</tr>
</thead>
<tbody>
<tr>
<td>P. UDP Header's Port Number</td>
<td>I. 48</td>
</tr>
<tr>
<td>Q. Ethernet MAC Address</td>
<td>II. 8</td>
</tr>
<tr>
<td>R. IPv6 Next Header</td>
<td>III. 32</td>
</tr>
<tr>
<td>S. TCP Header's Sequence Number</td>
<td>IV. 16</td>
</tr>
</tbody>
</table>

(A) P-III, Q-IV, R-II, S-I
(B) P-II, Q-I, R-IV, S-III
(C) P-IV, Q-I, R-II, S-III
(D) P-IV, Q-I, R-III, S-II
Key: (C)
Exp: The length of the Ethernet MAC address is 48 bits. So Q-I
The length of the TCP Header’s sequence number is 32 bits. So, S-III
The length of the UDP Header’s port number is 16 bits. So, P-IV
The length of the IPv6 Next header is 8 bits So, R-II
So, P-IV Q-I, S-III, R-II hence option ‘C’ correct.

15. Consider the following two tables and four queries in SQL.

Book (isbn, bname), Stock (isbn, copies)

Query 1: SELECT B.isbn, S.copies
FROM Book B INNER JOIN Stock S
ON B.isbn=S.isbn;

Query 2: SELECT B.isbn, S.copies
FROM Book B LEFT OUTER JOIN Stock S
ON B.isbn=S.isbn;

Query 3: SELECT B, isbn, S.copies
FROM Book B RIGHT OUTER JOIN Stock S
ON B.isbn=S.isbn;

Query 4: SELECT B, isbn, S.copies
FROM Book B FULL OUTER JOIN Stock S
ON B.isbn=S.isbn;

Which one of the queries above is certain to have an output that is a superset of the outputs of the other three queries?

(A) Query 1  (B) Query 2  (C) Query 3  (D) Query 4

Key: (D)
Exp: Full outer join result will always contain the result of inner, left outer and right outer joins because it will preserve the un-matched tuples of left side and right side tables after performing join.
Q1: inner join will not preserve both side un-matched tuples

<table>
<thead>
<tr>
<th>isbn</th>
<th>copies</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>30</td>
</tr>
</tbody>
</table>

Q2: Left outer join will preserve only left side table unmatched tuples

<table>
<thead>
<tr>
<th>isbn</th>
<th>copies</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>null</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>30</td>
</tr>
<tr>
<td>4</td>
<td>null</td>
</tr>
</tbody>
</table>

Q3: Right outer join will preserve only right side table unmatched tuples

<table>
<thead>
<tr>
<th>isbn</th>
<th>copies</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>30</td>
</tr>
<tr>
<td>5</td>
<td>null</td>
</tr>
</tbody>
</table>

Q4: full outer join will preserve both side tables un-matched tuples

<table>
<thead>
<tr>
<th>isbn</th>
<th>copies</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>null</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>30</td>
</tr>
<tr>
<td>4</td>
<td>null</td>
</tr>
<tr>
<td>5</td>
<td>20</td>
</tr>
</tbody>
</table>

16. A queue is implemented using a non-circular singly linked list. The queue has a head pointer and a tail pointer, as shown in the figure. Let \( n \) denote the number of nodes in the queue. Let enqueue be implemented by inserting a new node at the head and dequeue be implemented by deletion of a node from the tail.

Which one of the following is the time complexity of the most time-efficient implementation of enqueue and dequeue, respectively, for this data structure?

(A) \( \theta(1), \theta(1) \)  
(B) \( \theta(1), \theta(n) \)  
(C) \( \theta(n), \theta(1) \)  
(D) \( \theta(n), \theta(n) \)
15. Enqueue: operation can be performed by inserting a node at the beginning and modifying the head pointer to point to new node. This requires $\Theta(1)$ time.

Dequeue: operation needed to point to previous node of the node pointed by tail pointer that requires traversal of the node till the last node. This operation requires $\Theta(n)$ time in the worst case.

17. Let $G$ be a finite group on 84 elements. The size of a largest possible proper subgroup of $G$ is____.

Key: (42)

Exp: Lagrange’s theorem states that “the order of subgroup should divide the order of group”

Given that, group $G$ consists of 84 elements, so the factors of 84 are 1, 2, 3, 4, 6, 7, 12, 14, 21, 28, 42 and 84.

Since we known that $\text{Order of Subgroup}(L) \neq \text{Order of Group}(G)$, the size of a largest possible proper subgroup is 42.

18. The set of all recursively enumerable languages is

(A) closed under complementation.
(B) closed under intersection.
(C) a subset of the set of all recursive languages.
(D) an uncountable set.

Key: (B)

Exp: Option (A), Complement of Recursive Enumerable language is Non Recursive Enumerable language.

Option (B), Recursive Enumerable language are closed under complementation, hence TRUE

Option (C), Recursive Enumerable is not subset of recursive languages instead it’s a superset of recursive languages.

Option (D), Recursive languages are Turing recognizable and number of Turing machine is countable.

19. The chromatic number of the following graph is _____.

[Graph diagram]
Key: (3)

Exp: We can see that there are 3 mutually adjacent vertices (b, c, d) in the graph, so its chromatic number is at least 3. Further we can colour the vertices of the graph with 3 colours.

\[ X(G) = 3 \]

Alternate method:

Welch Powell's Theorem

\[
\begin{array}{ccccccc}
\text{b} & \text{c} & \text{d} & \text{e} & \text{f} & \text{a} \\
C_1 & C_2 & C_3 & C_1 & C_2 & C_3 \\
\end{array}
\]

\[ X(G) \leq 3 \text{ & } X(G) \geq 3 \quad (\because \text{we have 3 mutually adjacent vertices}) \]

\[ X(G) = 3 \]

20. A 32-bit wide main memory unit with a capacity of 1 GB is built using 256M × 4-bit DRAM chips. The number of rows of memory cells in the DRAM chip is \(2^{14}\). The time taken to perform one refresh operation is 50 nanoseconds. The refresh period is 2 milliseconds. The percentage (rounded to the closest integer) of the time available for performing the memory read/write operations in the main memory unit is_______

Key: (59)

Exp: Time for a refresh operation for a row is given as \(50 \times 10^{-9}\) sec

Time requires for the refreshing all the rows is \(2^{14} \times 50 \times 10^{-9}\) sec = 0.8192 millisecond

Periodic interval is 2 milliseconds

Overhead of the refreshing operation is \(= \frac{0.8192}{2} \times 100 = 40.96\%\)

The percentage (rounded to the closest integer) of the time available for performing the memory read write operations in the main memory unit is 59.04 ≈ 59%.

21. Which one of the following is a closed form expression for the generating function of the sequence \(\{a_n\}\), where \(a_n = 2n+3\) for all \(n = 0, 1, 2\ldots\)?

(A) \(\frac{3}{(1-x)^3}\)  \hspace{1cm} (B) \(\frac{3x}{(1-x)^3}\)  \hspace{1cm} (C) \(\frac{2-x}{(1-x)^2}\)  \hspace{1cm} (D) \(\frac{3-x}{(1-x)^2}\)
Key: (D)

Exp: The required series is [3, 5, 7, 9, 11………]

\[ a_n = 2n + 3 \]

∴ The required generating function will be given by

\[
\sum_{n=0}^{\infty} (2n+3)x^n = 2\sum_{n=0}^{\infty} nx^n + 3\sum_{n=0}^{\infty} x^n \\
= 2\frac{x}{(1-x)^2} + 3\frac{1}{1-x} = \frac{2x+3-3x}{(1-x)^2} = \frac{3-x}{(1-x)^2}
\]

22. Consider a system with 3 processes that share 4 instances of the same resource type. Each process can request a maximum of \( K \) instances. Resource instances can be requested and released only one at a time. The largest value of \( K \) that will always avoid deadlock is ____.

Key: (2)

Exp: Number of processes =3
Number of instances of same resource = 4.

∴ Demand, \(<\) no. of processes + no. of resources

\[ 3K < 3 + 4 \implies 3K < 7 \implies K < \frac{7}{3} \implies K < 2.3333 \implies K = 2 \]

23. Consider a process executing on an operating system that uses demand paging. The average time for a memory access in the system is \( M \) units if the corresponding memory page is available in memory and \( D \) units if the memory access causes a page fault. It has been experimentally measured that the average time taken for a memory access in the process is \( X \) units. Which one of the following is the correct expression for the page fault rate experienced by the process?

(A) \( \frac{D-M}{X-M} \)  
(B) \( \frac{X-M}{D-M} \)  
(C) \( \frac{D-X}{D-M} \)  
(D) \( \frac{X-M}{D-X} \)

Key: (B)

Exp: Given that

\( M = \) Memory Access time
\( D = \) Page fault service time (S)
\( P = \) Page fault rate
\( X = \) Effective Access Time

In demand paging, Effective Access Time (EAT) = \( P*S + (1-P)M \)
\[ \Rightarrow X = P^* D + (1 - P) M \]
\[ \Rightarrow X = P^* D + M - MP \]
\[ \Rightarrow X - M = P^* D - M^* P \]
\[ \Rightarrow X - M = P(D - M) \Rightarrow P = \frac{X - M}{D - M} \]

24. Two people P and Q decide to independently roll two identical dice, each with 6 faces numbered 1 to 6. The person with the lower number wins. In case of a tie, they roll the dice repeatedly until there is no tie. Define a trial as a throw of the dice by P and Q. Assume that all 6 numbers on each dice are equi-probable and that all trials are independent. The probability (rounded to 3 decimal places) that one of them wins on the third trial is ______.

**Key:** (0.023)

**Exp:** Probability that one of them wins = \( P(\text{tie}) \times P(\text{tie}) \times P(\text{notie}) \)

\[
= \frac{6}{36} \times \frac{6}{36} \times \frac{30}{36} = \frac{5}{216} = 0.023
\]

25. The value of \( \int_{0}^{\pi/16} x \cos(x^2) \, dx \) correct to three decimal places (assuming that \( \pi = 3.14 \)) is ____.

**Key:** (0.29)

**Exp:** Let \( I = \int_{0}^{\pi/16} x \cos(x^2) \, dx \)

Put \( x^2 = t \Rightarrow 2x \, dx = dt \Rightarrow x \, dx = \frac{dt}{2} \)

for \( x = 0 \Rightarrow t = 0 \)

\( x = \frac{\pi}{16} \Rightarrow t = \frac{\pi^2}{16} \)

\[ \therefore I = \int_{0}^{\pi^2/16} \cos t \, \frac{dt}{2} = \frac{1}{2} \left[ \sin \left( \frac{\pi^2}{16} \right) - \sin 0 \right] = 0.29 \]

26. Consider Guwahati (G) and Delhi (D) whose temperatures can be classified as high (H), medium (M) and low (L). Let \( P(H_G) \) denote the probability that Guwahati has high temperature. Similarly \( P(M_G) \) and \( P(L_G) \) denotes the probability of Guwahati having medium and low temperatures respectively. Similarly, we use \( P(H_D), P(M_D) \) and \( P(L_D) \) for Delhi.

The following table gives the conditional probabilities for Delhi's temperatures given Guwahati's temperature
Consider the first row in the table above. The first entry denotes that if Guwahati has high temperature \((H_G)\) then probability of Delhi also having a high temperature \((H_D)\) is 0.40 i.e., \(P(H_D | H_G) = 0.40\). Similarly, the next two entries are \(P(M_D | H_G) = 0.48\) and \(P(L_D | H_G) = 0.12\). Similarly for the other rows.

If it is known that \(P(H_G) = 0.2, P(M_G) = 0.5\) and \(P(L_G) = 0.3\), then the probability (correct to two decimal places) that Guwahati has high temperature given that Delhi has high temperature is ________.

**Key:** (0.6)

**Exp:** \(P(H_G | H_D) = ?\)

We have \(P(H_G | H_D) = \frac{P(H_G \cap H_D)}{P(H_D)}\)

Where \(P(H_G \cap H_D) = P(H_G) \times P(H_D | H_G)\)

\[= 0.2 \times 0.4 = 0.18\]


\(:\) By Total Probability

\[= (0.2 \times 0.4) + (0.5 \times 0.1) + (0.3 \times 0.01) = 0.133\]

\[\therefore P(H_G | H_D) = \frac{0.08}{0.133} \approx 0.6\]

27. Assume that multiplying a matrix \(G_1\) of dimension \(p \times q\) with another matrix \(G_2\) of dimension \(q \times r\) requires \(pqr\) scalar multiplications. Computing the product of \(n\) matrices \(G_1G_2G_3...G_n\) can be done by parenthesizing in different ways. Define \(G_iG_{i+1}\) as an explicitly computed pair for a given parenthesization if they are directly multiplied. For example, in the matrix multiplication chain \(G_1G_2G_3G_4G_5G_6\) using parenthesization \((G_1(G_2G_3))(G_4(G_5G_6))\), \(G_2G_3\) and \(G_5G_6\) are the only explicitly computed pairs.

Consider a matrix multiplication chain \(F_1F_2F_3F_4F_5\), where matrices \(F_1, F_2, F_3, F_4\) and \(F_5\) are of dimensions \(2 \times 25, 25 \times 3, 3 \times 16, 16 \times 1\) and \(1 \times 1000\) respectively, in the parenthesization of \(F_1F_2F_3F_4F_5\) that minimizes the total number of scalar multiplications, the explicitly computed pairs are

(A) \(F_1F_2\) and \(F_3F_4\) only
(B) \(F_2F_3\) only
(C) \(F_3F_4\) only
(D) \(F_1F_2\) and \(F_4F_5\) only

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Key: (C)

Exp: $F_1F_2F_3F_4F_5$

Cost of the multiplication is its explicit pairs are $F_1F_2$ AND $F_3F_4$ is $(((F_1F_2)(F_3F_4))F_5))$ = $F_1F_2 \times 150$

$F_3F_4 = 48$

$((F_1F_2)(F_3F_4)) = 6$

$(((F_1F_2)(F_3F_4))F_5)) = 2000$

Total = 2204

Cost of the multiplication is if its explicit pairs are $F_1F_2$ AND $F_3F_4$ is $((F_1(F_2(F_3F_4))))F_5$

$F_3F_4 = 48$

$F_2(F_3F_4) = 75$

$F_1(F_2(F_3F_4)) = 50$

$((F_1(F_2(F_3F_4)))F_5) = 2000$

Total = 2173

28. The instruction pipeline of a RISC processor has the following stages: Instruction Fetch (IF), Instruction Decode (ID), Operand Fetch (OF), Perform Operation (PO) and Write back (WB). The IF, ID, OF and WB stages take 1 clock cycle each for every instruction. Consider a sequence of 100 instructions. In the PO stage, 40 instructions take 3 clock cycles each, 35 instructions take 2 clock cycles each, and the remaining 25 instructions take 1 clock cycle each. Assume that there are no data hazards and no control hazards. The number of clock cycles required for completion of execution of the sequence of instructions is______.

Key: (219)

Exp:

<table>
<thead>
<tr>
<th>Number of instruction</th>
<th>Number of cycle required in Execute stage</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>3</td>
</tr>
<tr>
<td>35</td>
<td>2</td>
</tr>
<tr>
<td>25</td>
<td>1</td>
</tr>
</tbody>
</table>

Average number of cycles required is

$= .40 \times 3 + .35 \times 2 + .25 \times 1 = 2.15$ cycles

We can assume that pipeline is behaving as Instruction Fetch (IF), Instruction Decode (ID), Operand Fetch (OF), and Write back (WB) performed in 1 cycle and Perform Operation (PO) required 2.15 cycle.
On an average first instruction completed in 1+1+1+1+2.15 cycles = 6.15;
and rest 99 instruction completed in 99× 2.15 = 212.85 cycles
Total number of cycles are 6.15+212.85 = 219 cycles.

29. Consider the following solution to the producer-consumer synchronization problem. The shared buffer size is \( N \). Three semaphores *empty*, *full* and *mutex* are defined with respective initial values of 0, \( N \) and 1. Semaphore *empty* denotes the number of available slots in the buffer, for the consumer to read from. Semaphore *full* denotes the number of available slots in the buffer, for the producer to write to. The placeholder variables, denoted by \( P, Q, R, \) and \( S \), in the code below can be assigned either *empty* or *full*. The valid semaphore operations are: *wait* () and *signal* ()

<table>
<thead>
<tr>
<th>Producer:</th>
<th>Consumer:</th>
</tr>
</thead>
<tbody>
<tr>
<td>do {</td>
<td>do {</td>
</tr>
<tr>
<td>wait(P);</td>
<td>wait(R);</td>
</tr>
<tr>
<td>wait(mutex);</td>
<td>wait(mutex);</td>
</tr>
<tr>
<td>//Add item to buffer</td>
<td>//Consume item to buffer</td>
</tr>
<tr>
<td>signal(mutex);</td>
<td>signal(mutex);</td>
</tr>
<tr>
<td>signal(Q);</td>
<td>signal(S);</td>
</tr>
<tr>
<td>}while(1);</td>
<td>}while(1);</td>
</tr>
</tbody>
</table>

Which one of the following assignments to \( P, Q, R \) and \( S \) will yield the correct solution?

(A) \( P: \) full, \( Q: \) full, \( R: \) empty, \( S: \) empty
(B) \( P: \) empty, \( Q: \) empty, \( R: \) full, \( S: \) full
(C) \( P: \) full, \( Q: \) empty, \( R: \) empty, \( S: \) full
(D) \( P: \) empty, \( Q: \) full, \( R: \) full, \( S: \) empty

**Key:** (C)

**Exp:** Given that

\( N = \) Buffer size

No. of semaphores = 3  
\( (\)empty,  full,  mutex) \( ) \rightarrow \) initial values

\( 0 \)  \( N \)  \( 1 \)

\( \Rightarrow \) Semaphore “empty” denotes the no. of available slots in the Buffer for the consumer to read from

\( \Rightarrow \) Semaphore “full” denotes the no. of available slots in the buffer for the producer to write to.
Wait (P); Wait(R); Wait(mutex); Wait(mutex); //Add item to buffer Signal(mutex); Signal(mutex); Signal(Q); Signal(S); } while (1); } while(1); With option C, only producer can produce item into buffer first and then consumer can start consuming that placed item from buffer.

Option D and B are wrong because producer can’t produce any item but consumer can consume an item from buffer which is impractical.

With option A, a producer can do two successive operations and consumer never consume. This is also impractical.

30. A lexical analyzer uses the following patterns to recognize three tokens $T_1$, $T_2$, and $T_3$ over the alphabet \{a, b, c\}.

$T_1$: \texttt{a?\((b|c)\)*a}

$T_2$: \texttt{b?\((a|c)\)*b}

$T_3$: \texttt{c?\((b|a)\)*c}

Note that ‘\?' means 0 or 1 occurrence of the symbol ‘$x$’. Note also that the analyzer outputs the token that matches the longest possible prefix.

If the string \textit{bbaacabc} is processed by the analyzer, which one of the following is the sequence of tokens it outputs?

(A) $T_1T_2T_3$  
(B) $T_1T_3$  
(C) $T_2T_1T_3$  
(D) $T_3T_3$

**Key:** (D)

**Exp:** $T_3T_3$; Given string ‘bbaacabc’

The longest string can be generated by $T_3$ is ‘bbaac’

$c?(b|a)*c$

zero occurrence of $c$, ‘bbaa’ can be generated by middle part and we can end with $c$.

The remaining string $abc$ can be generated $T_3$

31. Consider a matrix $P$ whose only eigenvectors are the multiplies of $\begin{bmatrix} 1 \\ 4 \end{bmatrix}$.

Consider the following statements:

(I) $P$ does not have an inverse

(II) $P$ has a repeated eigenvalue
(III) P cannot be diagonalized
Which one of the following options is correct?
(A) Only I and III are necessarily true
(B) Only II is necessarily true
(C) Only I and II are necessarily true
(D) Only II and III are necessarily true

Key: (D)

Exp: Given Eigen vectors of P are multiples of \[
\begin{bmatrix}
1 \\
4
\end{bmatrix}
\]
\[P\text{ is a } 2 \times 2 \text{ matrix with repeated Eigen value } \& P \text{ cannot be diagonalized since } P \text{ has dependent Eigen vectors}
\]
\[\therefore \text{ I & III are necessarily true}
\]

32. A processor has 16 integer registers (R0, R1….R15) and 64 floating point registers (F0,F1…….F63). It uses a 2-byte instruction format. There are four categories of instructions: Type-1, Type-2, Type-3, and Type-4. Type-1 category consists of four instructions, each with 3 integer register operands (3Rs). Type-2 category consists of eight instructions, each with 2 floating point register operands (2Fs). Type-3 category consists of fourteen instructions, each with one integer register operand and one floating point register operand (1R+1F). Type-4 category consists of X instructions, each with a floating point register operand (IF). The maximum value of N is_______.

Key: (32)

Exp: Type-1 category consists of four instructions, each with 3 integer register operands (3Rs).

\[
\begin{align*}
&\text{16 bits} \\
&\begin{array}{cccc}
&\text{4 instructions} & \text{4 bits} & \text{4 bits} & \text{4 bits}
\end{array}
\end{align*}
\]

Type-2 category consists of eight instructions, each with 2 floating point register operands (2Fs)

\[
\begin{align*}
&\text{16 bits} \\
&\begin{array}{ccc}
&\text{8 instructions} & \text{6 bits} & \text{6 bits}
\end{array}
\end{align*}
\]

Type-3 category consists of fourteen instructions, each with one integer register operand and one floating point register operand (1R+1F).

\[
\begin{align*}
&\text{16 bits} \\
&\begin{array}{cc}
&\text{14 instructions} & \text{4 bits} & \text{6 bits}
\end{array}
\end{align*}
\]
Type-4 category consists of X instructions, each with a floating point register operand (IF).

<table>
<thead>
<tr>
<th>X instructions</th>
<th>16 bits</th>
<th>6 bits</th>
</tr>
</thead>
</table>

Total number of instruction possible $2^{10}$, but we need to discard bits used in Type-I, Type-II, Type-III, and Type-IV

$$=2^{10} - (4 \times 2^6) - (8 \times 2^6) - (14\times 2^4)$$

$$=256 - 224 = 32$$

33. Consider the following parse tree for the expression $a#b$c$d#e#f$, involving two binary operators $\#$ and $\$$.

Which one of the following is correct for the given parse tree?

(A) $\$ has higher precedence and is left associative; $\#$ is right associative

(B) $\#$ has higher precedence and is left associative; $\$ is right associative

(C) $\$ has higher precedence and is left associative; $\#$ is left associative

(D) $\#$ has higher precedence and is right associative; $\$ is left associative

**Key:** (A)

**Exp:**
The given expression $a#b$c$d#e#f$

d is associated with operator $\$ not $\#$ hence $\$ is having higher precedence than $\#$
The sub expression $b$c$d$, in this $c$ is associated with operator on the right hand side hence $\$ is left associative.

After evaluation of $b$c$d$, rest of the expression will be evaluated from right e.g. $e#f$ will be executed first hence right associative.

Hence $\$ has higher precedence and is left associative; $\#$ is right associative
34. Consider the minterm list form of a Boolean function $F$ given below:

$$F(P, Q, R, S) = \sum m(0, 2, 5, 7, 9, 11) + d(3, 8, 10, 12, 14)$$

Here, $m$ denotes a minterm and $d$ denotes a don't care term. The number of essential prime implicants of the function $F$ is _______

Key: (3)

Exp: $F(P, Q, R, S) = \sum m(0, 2, 5, 7, 9, 11) + d(3, 8, 10, 12, 14)$

![K-map diagram]

EPI’s are those groups, which contains at least one minterm which is not shared by multiple groups. In the above k-map there are 3 such groups and hence total 3 EPIs are there.

35. Consider the relations $r(A, B)$ and $s(B, C)$, where $s.B$ is a primary key and $r.B$ is a foreign key referencing $s.B$. Consider the query

$$Q: r \sigma_{r < s} (s)$$

Let LOJ denote the natural left outer-join operation. Assume that $r$ and $s$ contain no null values. Which one of the following queries is NOT equivalent to $Q$?

(A) $\sigma_{r < s} (r \bowtie s)$  
(B) $\sigma_{B<s} (r \text{LOJs})$  
(C) $r \text{LOJ}(\sigma_{B<s} (s))$  
(D) $\sigma_{B<s} (r) \text{LOJs}$

Key: (C)

Exp:

![Relation diagram]

$$Q: r \bowtie (\sigma_{B<s} (s))$$

Here first filter the tuples of $s$ in such a way that all the tuples which are having $B$ value greater than or equal to 5 are eliminated and then joined with $r$. So the output will be $A$, $B$, $C$ combination where $B$ value is less than 5.
Ex:

\[
\begin{array}{ccc}
\text{r} & 1 & 2 \\
& 2 & 3 \\
& 4 & 5 \\
& 5 & 2 \\
\end{array}
\quad
\begin{array}{ccc}
\text{s} & 2 & 3 \\
& 6 & 4 \\
& 5 & 6 \\
& 3 & 7 \\
\end{array}
\]

Q: \( r \Join (\sigma_{B<5}(s)) \) will give output

\[
\begin{array}{ccc}
A & B & C \\
1 & 2 & 3 \\
2 & 3 & 7 \\
5 & 2 & 3 \\
\end{array}
\]

Options A, B and D will generate the output which is same as the given query

Option (C) output is

\[
\begin{array}{ccc}
A & B & C \\
1 & 2 & 3 \\
2 & 3 & 7 \\
4 & 5 & \text{null} \\
5 & 2 & 3 \\
\end{array}
\]

36. Consider the following C code. Assume that unsigned long int type length is 64 bits.

\[
\text{unsigned long int fun(unsigned long int n)}
\end{array}
\]

\[
\text{unsigned long int i, j = 0, sum = 0;}
\end{array}
\]

\[
\text{for } (i = n; i > 1; i = i/2) \quad j++;
\end{array}
\]

\[
\text{for } (j > 1; j = j/2) \quad \text{sum++;}
\end{array}
\]

\[
\text{return(sum);}
\end{array}
\]

The value returned when we call fun with the input \(2^{40}\) is

(A) 4 \hspace{1cm} (B) 5 \hspace{1cm} (C) 6 \hspace{1cm} (D) 40

Key: (B)

EXP: for \( (i = n; i > 1; i = i/2) \)

\[
\text{j++}
\]

<table>
<thead>
<tr>
<th>Condition</th>
<th>Increment of j</th>
</tr>
</thead>
<tbody>
<tr>
<td>(2^{40}&gt;1)</td>
<td>1</td>
</tr>
<tr>
<td>(2^{39}&gt;1)</td>
<td>2</td>
</tr>
<tr>
<td>(2^{38}&gt;1)</td>
<td>3</td>
</tr>
</tbody>
</table>
Final value of \( j \) is 40

**Second for loop**

```cpp
for(; j > 1;  j = j/2)
    sum++;
```

<table>
<thead>
<tr>
<th>Condition</th>
<th>Increment of ( \text{sum} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>40 &gt; 1</td>
<td>1</td>
</tr>
<tr>
<td>20 &gt; 1</td>
<td>2</td>
</tr>
<tr>
<td>10 &gt; 1</td>
<td>3</td>
</tr>
<tr>
<td>5  &gt; 1</td>
<td>4</td>
</tr>
<tr>
<td>2  &gt; 1</td>
<td>5</td>
</tr>
<tr>
<td>1  &gt; 1</td>
<td>Break</td>
</tr>
</tbody>
</table>

Final value of \( \text{sum} \) is 5

37. The size of the physical address space of a processor is \( 2^P \) bytes. The word length is \( 2^W \) bytes. The capacity of cache memory is \( 2^N \) bytes. The size of each cache block is \( 2^M \) words. For a \( K \)-way set-associative cache memory, the length (in number of bits) of the tag field is

(A) \( P-N-\log_2 K \)  
(B) \( P-N+\log_2 K \)  
(C) \( P-N-M-W-\log_2 K \)  
(D) \( P-N-M-W+\log_2 K \)

**Key:** (B)

**Exp:** Physical address space is \( 2^P \) bits, hence we have \( P \) bits in physical address space.

<table>
<thead>
<tr>
<th>TAG</th>
<th>Set offset</th>
<th>Byte offset</th>
</tr>
</thead>
</table>

Word length is given as \( 2^W \) bytes  
Capacity is given as \( 2^N \) bytes  
Size of the block is \( 2^M \) words  
Size of the cache block is \( 2^M \times 2^W \) bytes = \( 2^{M+W} \) bytes  
Byte offset will be \( M+W \) bits
Numbers of cache blocks are \( \frac{2^N}{2^{M+W}} = 2^{N-M-W} \)

Numbers of sets are \( \frac{\text{Total cache blocks}}{K} = \frac{2^{N-M-W}}{K} \)

Number of bits for set offset = \( \log_2 \frac{2^{N-M-W}}{K} = N - M - W - \log_2 K \)

Choose a value for \( x \) that will maximize the number of minimum weight (MWSTs) of \( G \). The number of MWSTs of \( G \) for this value of \( x \) is_______

Key: (4)

Exp: For \( x = 1, 2, 3, 4 \) only 2 spanning trees possible
for \( x = 5 \) total 4 spanning trees possible
39. Given a language $L$, define $L^i$ as follows:

- $L^0 = \{ \varepsilon \}$
- $L^i = L^{i-1}$ for all $i > 0$

The order of a language $L$ is defined as the smallest $k$ such that $L^k = L^{k+1}$.

Consider the language $L_1$ (over alphabet 0) accepted by the following automaton.

The order of $L_1$ is

**Key:** (2)

**Exp:** The language accepted by the machine is $L_1 = \{ \varepsilon, 0, 000, 00000 \ldots \}$; which is nothing but zero and odd occurrences of 0’s

$L_1^2 = \{ \varepsilon, 0, 000, 00000, \ldots \ldots \} = \{ \varepsilon, 0, 00, 0000, \ldots \ldots \}$ which is 0*

For any repetition of $L_1$, 2 or more time of it will always contains 0*

Hence answer is 2

$L_1^2 = L_1^3$. $L$ is **TRUE**

40. Let $N$ be the set of natural number

Consider the following sets

- $P$: Set of Rational numbers (positive and negative)
- $Q$: Set of functions from $\{0, 1\}$ to $N$
- $R$: Set of functions from $N$ to $\{0, 1\}$
- $S$: Set of finite subset of $N$

Which of the sets above are countable?

(A) $Q$ and $S$ only  (B) $P$ and $S$ only  (C) $P$ and $R$ only  (D) $P$, $Q$ and $S$ only
Key: (D)

Exp: Set of rational number can be represented in \( \frac{P}{Q} \) format are countable since one to one correspondence exists

Set of functions from \{0, 1\} to \( N \) will be equal to \( N^2 \) which is also countable.
Set of functions from \( N \) to \{0, 1\} will be \( 2^N \) which is uncountable
Set of finite subset of \( N \) will always be countable.

41. Consider a storage disk with 4 platters (numbered as 0.1. 2 and 3), 200 cylinders (numbered as 0. 1. .... 199), and 256 sectors per track (numbered as 0. 1. ... 255). The following 6 disk requests of the form [sector number. cylinder number. platter number] are received by the disk controller at the same time:

\[ [120, 72, 2], [180, 134, 1], [60, 20, 0], [212, 86, 3], [56, 116, 2], [118, 16, 1] \]

Currently the head is positioned at sector number 100 of cylinder 80. and is moving: towards higher cylinder numbers. The average power dissipation in moving the head over 100 cylinders is 20 milliwatts and for reversing the direction of the head movement once is 15 milliwatts. Power dissipation associated with rotational latency and switching of head between different planers is negligible.

The total power consumption in milliwatts to satisfy all of the above disk requests using the Shortest Seek Time First disk scheduling algorithm is \[ \boxed{85} \].

Key: (85)

Exp: No. of head direction changes =3
Power dissipation for each direction change = 15 milliwatts.
Total Power dissipation for each direction change = \( 3 \times 15 = 45 \) milliwatts.
By ignoring power dissipation for switching of head between different platters \( \Rightarrow \)
\[ \text{Total head movements} = |80 - 86| + |86 - 72| + |72 - 134| + |134 - 16| \\
= 6 + 14 + 62 + 118 = 200 \]

Current Head position
\[ \downarrow \]
16 20 72 80 86 116 134

Direction change (1)

Direction change (2)

Direction change (3)

Power dissipation /100 cylinders = 20 milliwatts
Power dissipation /200 cylinders = 40 milliwatts
The Total Power Dissipation = 40+45 = 85 milliwatts

42. Consider the following program written in pseudo-code. Assume that \( x \) and \( y \) are integers.

\[
\text{Count}(x,y) \begin{cases} 
\text{if } (y \neq 1) \{ \\
\quad \text{if } (x \neq 1) \{ \\
\quad\quad \text{printf} \left( \ast \right); \\
\quad\quad \text{Count} \left( \frac{x}{2}, y \right); \\
\quad\} \\
\quad \text{else} \{ \\
\quad\quad y = y - 1; \\
\quad\quad \text{Count} (1024, y); \\
\quad\} \\
\} 
\end{cases}
\]

The number of times that the print statement is executed by the call \text{Count}(1024, 1024) is ____.

Key: (10230)

Exp: The call for the function \text{Count}(1024, 1024) will be executed as following

<table>
<thead>
<tr>
<th>Function</th>
<th>Print statement</th>
</tr>
</thead>
<tbody>
<tr>
<td>\text{Count}(1024, 1024)</td>
<td>1</td>
</tr>
<tr>
<td>\text{Count}(512, 1024)</td>
<td>1</td>
</tr>
<tr>
<td>\text{Count}(256, 1024)</td>
<td>1</td>
</tr>
<tr>
<td>\text{Count}(128, 1024)</td>
<td>1</td>
</tr>
<tr>
<td>\text{Count}(64, 1024)</td>
<td>1</td>
</tr>
<tr>
<td>Count(32, 1024)</td>
<td>1</td>
</tr>
<tr>
<td>Count(16, 1024)</td>
<td>1</td>
</tr>
<tr>
<td>Count(8, 1024)</td>
<td>1</td>
</tr>
<tr>
<td>Count(4, 1024)</td>
<td>1</td>
</tr>
<tr>
<td>Count(2, 1024)</td>
<td>1</td>
</tr>
</tbody>
</table>

Total 10 print statements will be printed till x reaches to 1
For call Count(1, 1024)
Count(1024, 1023) will be executed hence

<table>
<thead>
<tr>
<th>Function</th>
<th>Print statement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Count(1024, 1024)</td>
<td>10</td>
</tr>
<tr>
<td>Count(1024, 1023)</td>
<td>10</td>
</tr>
<tr>
<td>.</td>
<td>.</td>
</tr>
<tr>
<td>.</td>
<td>.</td>
</tr>
<tr>
<td>.</td>
<td>.</td>
</tr>
<tr>
<td>Count(2, 1024)</td>
<td>10</td>
</tr>
<tr>
<td>Total print</td>
<td>1023 x 10 = 10230</td>
</tr>
</tbody>
</table>

43. Consider the following problems. \( L(G) \) denotes the language generated by a grammar \( G \). \( L(M) \) denotes the language accepted by a machine \( M \).
(I) For an unrestricted grammar \( G \) and a string \( w \) whether \( w \in L(G) \)
(II) Given a Turing machine \( M \), whether \( L(M) \) is regular
(III) Given two grammars \( G_1 \) and \( G_2 \), whether \( L(G_1) = L(G_2) \)
(IV) Given an NFA \( N \), whether there is a deterministic PDA \( P \) such that \( N \) and \( P \) accept the same language.
Which one of the following statements is correct?
(A) Only I and II are undecidable
(B) Only III is undecidable
(C) Only II and IV are undecidable
(D) Only I, II and III are undecidable

Key: (D)

Exp: (I) is a membership problem for Turing machine and is undecidable
(II) According to RICE theorem any language property of Turing machine is undecidable.
(III) Given two grammars \( G_1 \) and \( G_2 \), whether \( L(G_1) = L(G_2) \) is an equivalence problem and undecidable for random grammar.

44. Consider an IP packet with a length of 4,500 bytes that includes a 20-byte IPv4 header and a 40-byte TCP header. The packet is forwarded to an IPv4 router that supports a Maximum Transmission Unit (MTU) of 600 bytes. Assume that the length of the IP header in all the outgoing fragments of this packet is 20 bytes. Assume that the fragmentation offset value stored in the first fragment is 0.
The fragmentation offset value stored in the third fragment is ____________.
45. Consider the first-order logic sentence

\[ \varphi \equiv \exists x \exists y \forall u \forall v \forall w \forall x \forall y \, \psi(s, t, u, v, w, x, y) \]

Where \( \psi(s, t, u, v, w, x, y) \) is a quantifier-free first-order logic formula using only predicate symbols and possibly equality, but no function symbols. Suppose \( \varphi \) has a model with a universe containing 7 elements.

Which one of the following statements is necessarily true?
(A) There exists at least one model of \( \varphi \) with universe of size less than or equal to 3.
(B) There exists no model of \( \varphi \) with universe of size less than or equal to 3.
(C) There exists no model of \( \varphi \) with universe of size greater than 7.
(D) Every model of \( \varphi \) has a universe of size equal to 7.

Key: (A)

46. Let \( G \) be a simple undirected graph. Let \( T_D \) be a depth first search tree of \( G \). Let \( T_B \) be a breadth first search tree of \( G \). Consider the following statements.
(I) No edge of \( G \) is a cross edge with respect to \( T_D \) (A cross edge in \( G \) is between two nodes neither of which is an ancestor of the other in \( T_D \)).
(II) For every edge \((u, v)\) of \( G \), if \( u \) is at depth \( i \) and \( v \) is at depth \( j \) in \( T_B \), then \(|i - j| = 1\).

Which of the statements above must necessarily be true?
(A) I only
(C) Both I and II
(B) II only
(D) Neither I nor II
47. In a system, there are three types of resources: $E$, $F$ and $G$. Four processes $P_0$, $P_1$, $P_2$ and $P_3$ execute concurrently. At the outset, the processes have declared their maximum resource requirements using a matrix named Max as given below. For example, $\text{Max}(P_1, F)$ is the maximum number of instances of $F$ that $P_2$ would require. The number of instances of the resources allocated to the various processes at any given state is given by a matrix named Allocation.

Consider a state of the system with the Allocation matrix as shown below, and in which 3 instances of $E$ and 3 instances of $F$ are the only resources available.

<table>
<thead>
<tr>
<th>Allocation</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$E$</td>
</tr>
<tr>
<td>$P_0$</td>
<td>1</td>
</tr>
<tr>
<td>$P_1$</td>
<td>1</td>
</tr>
<tr>
<td>$P_2$</td>
<td>1</td>
</tr>
<tr>
<td>$P_3$</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>5</td>
</tr>
</tbody>
</table>

From the perspective of deadlock avoidance, which one of the following is true?

(A) The system is in safe state.
(B) The system is not in safe state, but would be safe if one more instance of $E$ were available.
(C) The system is not in safe state, but would be safe if one more instance of $F$ were available.
(D) The system is not in safe state, but would be safe if one more instance of $G$ were available.

Key: (A)

Exp:

\[
\begin{align*}
\text{Allocation} & \quad \text{Max} & \quad \text{Need} = (\text{Max} - \text{Alloc}) & \quad \text{Avail} \\
& E \quad F \quad G & E \quad F \quad G & E \quad F \quad G \\
\text{P}_0 & 1 \quad 0 \quad 1 & 4 \quad 3 \quad 1 & 3 \quad 3 \quad 0 & 3 \quad 3 \quad 0 \\
\text{P}_1 & 1 \quad 1 \quad 2 & 2 \quad 1 \quad 4 & 1 \quad 0 \quad 2 \\
\text{P}_2 & 1 \quad 0 \quad 3 & 1 \quad 3 \quad 3 & 0 \quad 3 \quad 0 \\
\text{P}_3 & 2 \quad 0 \quad 0 & 5 \quad 4 \quad 1 & 3 \quad 4 \quad 1 \\
\end{align*}
\]
<table>
<thead>
<tr>
<th></th>
<th>P₀</th>
<th>P₂</th>
<th>P₁</th>
<th>P₃</th>
</tr>
</thead>
<tbody>
<tr>
<td>Need</td>
<td>≤ Available</td>
<td>Need</td>
<td>≤ Available</td>
<td>Need</td>
</tr>
<tr>
<td>(330)</td>
<td>≤ (330)</td>
<td>(030)</td>
<td>≤ (431)</td>
<td>(102)</td>
</tr>
<tr>
<td>TRUE</td>
<td></td>
<td>TRUE</td>
<td></td>
<td>TRUE</td>
</tr>
</tbody>
</table>

:. Work = Work + Allocation

= (330) + (101)

= (431) + (103)

= (534) + (112)

= (646)

= (846)

safe sequence \(-\text{I}\)

\(< P₀ P₂ P₁ P₃ >\)

Since, we are able to generate safe sequence; we can say the system is in safe state. Hence, the option A is correct.

Note: we can generate multiple safe sequences, not only the given one.

48. Let \(G\) be a graph with 100! Vertices, with each vertex labelled by a distinct permutation of the numbers 1, 2, ..., 100. There is an edge between vertices \(u\) and \(v\) if and only if the label of \(u\) can be obtained by swapping two adjacent numbers in the label of \(v\). Let \(y\) denote the degree of a vertex in \(G\) and \(z\) denote the number of connected components in \(G\).

Then \(y + 10z = \text{__________}\).

**Key: (109)**

**Exp:** If there are \(n\) numbers then we will have \((n-1)\) adjacent pairs. Therefore according to the given data every vertex adjacent to 99 vertices (because for 100 number sequences, we will have 99 adjacent pairs of numbers) and more over we will have only one component in the graph.

:. \(y = 99\) & \(z = 2\)

:. \(y + 10z = 99 + 10 = 109\)

49. Consider a simple communication system where multiple nodes are connected by a shared broadcast medium (like Ethernet or wireless). The nodes in the system use the following carrier-sense based medium access protocol. A node that receives a packet to transmit will carrier-sense the medium for 5 units of time. If the node does not detect any other transmission in this duration it starts transmitting its packet in the next time unit. If the node detects another transmission it waits until this other transmission finishes, and then begins to carrier-sense for 5 time units again. Once they start to transmit nodes do not perform any collision detection and continue transmission even if a collision occurs. All transmission last for 20 units of time. Assume that the transmission signal travels at the speed of 10 meters per unit time in the medium.

Assume that the system has two nodes P and Q, located at a distance \(d\) meters from each other. P starts transmitting a packet at time \(t=O\) after successfully completing its carrier-sense phase. Node Q has a packet to transmit at time \(r=O\) and begins to carrier-sense the medium.
The maximum distance $d$ (in meters, rounded to the closest integer) that allows Q to successfully avoid a collision between its proposed transmission and P's ongoing transmission is ____________.

**Key:** (50)

**Exp:** As per the question,

"A node that receives a packet to transmit with carrier sense the medium for 5 units of time which implies that

→ channel sensing time = 5 units of the propagation speed = 10 meters/unit time

In order to collision avoidance, P's packet have to be travelled a distance of 50 meters in 5 units of time.

(∵ P starts transmitting a packet at time $t=0$ after successfully completing its carrier-sense phase.
Node Q has a packet to transmit at time $r=0$ and begins to carrier-sense the medium.)

50. Consider the following languages:
I. $\{a^m b^n c^p d^q | m+p = n + q, \text{ where } m, n, p, q \geq 0\}$
II. $\{a^m b^n c^p d^q | m = n \text{ and } p = q, \text{ where } m, n, p, q \geq 0\}$
III. $\{a^m b^n c^p d^q | m = n = p \text{ and } p \neq q, \text{ where } m, n, p, q \geq 0\}$
IV. $\{a^m b^n c^p d^q | mn = p + q, \text{ where } m, n, p, q \geq 0\}$

Which of the languages above are context-free?
(A) I and IV only
(B) I and II only
(C) II and III only
(D) II and IV only

**Key:** (B)

**Exp:**
I. $\{a^m b^n c^p d^q | m+p = n + q, \text{ where } m, n, p, q \geq 0\}$ is a context free language.

The grammar is

$S \rightarrow aSd | PQR$
$P \rightarrow aPb | \epsilon$
$Q \rightarrow bQc | \epsilon$
$R \rightarrow cRd | \epsilon$

II. $\{a^m b^n c^p d^q | m = n \text{ and } p = q, \text{ where } m, n, p, q \geq 0\}$ is also context free language.

The grammar is

$S \rightarrow AB$
$A \rightarrow aAb | \epsilon$
$B \rightarrow cBd | \epsilon$

III, IV are not context free languages.
51. Consider the following four relational schemas. For each schema all non-trivial functional dependencies are listed. The underlined attributes are the respective primary keys.

Schema I: \(\text{Registration (rollno, courses)}\)
Field 'courses' is a set-valued attribute containing the set of courses a student has registered for:
Non-trivial functional dependency:
\(\text{Rollno} \rightarrow \text{courses}\)

Schema II: \(\text{Registration (rollno, courseid, email)}\)
Non-trivial functional dependency:
\(\text{rollno, courseid} \rightarrow \text{email}\)
\(\text{email} \rightarrow \text{rollno}\)

Schema III: \(\text{Registration (rollno, courseid, marks, grade)}\)
Non-trivial functional dependencies:
\(\text{rollno, courseid} \rightarrow \text{marks, grade}\)
\(\text{marks} \rightarrow \text{grade}\)

Schema IV: \(\text{Registration (rollno, courseid, credit)}\)
Non-trivial functional dependencies:
\(\text{rollno, courseid} \rightarrow \text{credit}\)
\(\text{courseid} \rightarrow \text{credit}\)

**Key:** (B)

**Exp:**
- Schema I: Candidate keys: \{Roll No\} and highest normal form is BCNF
- Schema II: Candidate keys are \{Rollno.courseid\} & \{email courseid\}
  3NF but not BCNF became the FD email \(\rightarrow\) roll no. violating BCNF condition
- Schema III: Candidate key \{roll no, courseid\}
  Highest normal form is 2NF it is not in 3NF because; the FD marks \(\rightarrow\) grade is a transitive dependency
- Schema IV: Candidate key \{roll no, courseid\} highest normal form is 1NF and it is not in 2NF, because the FD courseid \(\Rightarrow\) credit is a poortial dependency
52. Consider the following C program:

```c
#include<stdio.h>

void fun1(char *s1, char *s2){
    char *tmp;
    tmp = s1;
    s1 = s2;
    s2 = tmp;
}

void fun2(char **s1, char **s2){
    char *tmp;
    tmp = *s1;
    *s1 = *s2;
    *s2 = tmp;
}

int main (){  
    char *str1 = "Hi", *str2 = "Bye";
    fun1(str1, str2);  printf("%s %s ", str1, str2);
    fun2(&str1, &str2);  printf("%s %s", str1, str2);
    return 0;
}
```

The output of the program above is

(A) Hi Bye Bye Hi  (B) Hi Bye Hi Bye
(C) Bye Hi Hi Bye  (D) Bye Hi Bye Hi

**Key:** (A)

**Exp:**

`Fun1` is sending address of String “Hi” and “Bye” as parameter and stored in character pointer `S1` and `S2`.

`Fun1` swap the address hold by `S1` and `S2` and after performing the swap operation `S1` will hold address of string “Bye” and `S2` will hold address of “Hi”.

So effectively no swapping is done for `str1` and `str2` hence print value will be Hi bye.

Where in case of `fun2` address of character pointer is sent so its required to store them in double character pointer. In this we are swapping `str1` and `str2` through double pointer, hence `str1` will now hold address of “Bye” and `str2` will hold address of “Hi”

Print statement will print Bye Hi

53. The number of possible min-heaps containing each value from \{1,2,3,4, 5, 6, 7\} exactly once is ______.

**Key:** (80)
Exp:

First possible min heap is

Place I, II, III and IV can be filled with 4, 5, 6 and 7 with 4 ways = 24

Second possible min heap is

Place I, II, III and IV can be filled with 4, 5, 6 and 7 with 4 ways = 24

Third possible min heap is

Place I, II, can be placed with 3 and rest 3 position can be filled in 3 ways, hence total 2 × 3 × 2 × 1 = 12 ways

Fourth possible min heap is

Place III, IV, can be placed with 3 and rest 3 position can be filled in 3 ways, hence total 2 × 3 × 2 × 1 = 12 ways

6, 7 can arrange in two ways in position III and IV
6, 7 can arrange in 2 ways in position I and II

Total number of ways will be equal to 24+24+24+2+2+2+2=80

54. Consider the weights and values of items listed below. Note that there is only one unit of each item.

<table>
<thead>
<tr>
<th>Item number</th>
<th>Weight (in Kgs)</th>
<th>Value (in Rupees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>60</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
<td>28</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>20</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>24</td>
</tr>
</tbody>
</table>

The task is to pick a subset of these items such that their total weight is no more than 11 Kgs and their total value is maximized. Moreover, no item may be split. The total value of items picked by an optimal algorithm is denoted by $V_{opt}$. A greedy algorithm sorts the items by their value-to-weight ratios in descending order and packs them greedily, starting from the first item in the ordered list. The total value of items picked by the greedy algorithm is denoted $V_{greedy}$.

The value of $V_{opt} - V_{greedy}$ is____________.

**Key:** (16)

**Exp:** Applying the greedy method sorting them in descending order

<table>
<thead>
<tr>
<th>Item number</th>
<th>Weight (in Kgs)</th>
<th>Value (in Rupees)</th>
<th>value-to-weight ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>2</td>
<td>24</td>
<td>12</td>
</tr>
<tr>
<td>1</td>
<td>10</td>
<td>60</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>20</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
<td>28</td>
<td>4</td>
</tr>
</tbody>
</table>
Greedy method will pick first item 4 and earn the profit of 24 and fill the weight 2Kg
Remaining weight is 9 hence item 1 can’t be picked since it is having weight 10.
It will pick next item in the list which weight 4kg and profit 20 will be earned,
Total weight is 6 and profit is 44.
Remaining weight is 5 which can’t be filled by using item number 2.
\( V_{\text{greedy}} = 44 \)
Optimal solution is item 1 which weight 10Kg and profit 60.
\( V_{\text{opt}} - V_{\text{greedy}} = 60 - 40 = 16 \)

55. Consider the unsigned 8-bit fixed point binary number representation below.

\[ b_7 \, b_6 \, b_5 \, b_4 \, b_3 \, b_2 \, b_1 \, b_0 \]

Where the position of the binary point is between \( b_3 \) and \( b_2 \). Assume \( b_7 \) is the most significant bit.
Some of the decimal numbers listed below cannot be represented exactly in the above representation:
(i) 31.500  (ii) 0.875  (iii) 12.100  (iv) 3.001
Which one of the following statements is true?
(A) None of (i), (ii), (iii), (iv) can be exactly represented
(B) Only (ii) cannot be exactly represented
(C) Only (iii) and (iv) cannot be exactly represented
(D) Only (i) and (ii) cannot be exactly represented

Key: (C)

Exp:

(i) \[ [31.5]_{10} = [\overline{11111.1} \, 0]_2 \]

\[
\begin{array}{cccccc}
2^31 & 0.5 \\
2^15 - 1 & \times 2 \\
2^7 - 1 & 0 \quad 0 \\
2^3 - 1 & 1 \quad 2 \\
1 - 1 & 0 \quad 0
\end{array}
\]

(ii) \[ [0.875]_{10} = [\overline{0.111}]_2 \]

\[
\begin{array}{cccccc}
1 \quad 0.750 \\
0 \quad 0.1500 \\
1 \quad 0.000
\end{array}
\]
(iii) \[12.100\]_10 = [1100.0001100....]_2

\[
\begin{array}{c|c}
2 & 12 \\
2 & 6 - 0 \\
2 & 3 - 0 \\
& 1 - 1 \\
\hline
0 & 0.100 \\
& \times 2 \\
0 & 0.200 \\
& \times 2 \\
0 & 0.400 \\
& \times 2 \\
0 & 0.800 \\
& \times 2 \\
0 & 1.600 \\
& \times 2 \\
1 & 0.400 \\
\end{array}
\]

So, we can say we can't represent the decimal number 12.100 in its equivalent binary form, since the fractional part needs more than 3 bits.

(iv) \[3.001\]_10 = [11.00....]_2

\[
\begin{array}{c|c}
2 & 3 \\
& 1 - 1 \\
\hline
0 & 0.001 \\
& \times 2 \\
0 & 0.002 \\
& \times 2 \\
0 & 0.004 \\
& \times 2 \\
0 & 0.008 \\
& \times 2 \\
0 & 0.016 \\
\end{array}
\]

So, we can say we can't represent the decimal number 3.001 in its exact binary form, because the fractional part needs more than 3 bits.

Hence, we can have exact binary equivalent of (i) and (ii) but can't have exact binary equivalent of (iii) and (iv). Hence option (C) is correct.
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