Conventional Paper-I-2011

1. (a) Distinguish between direct and indirect band-gap materials with suitable r-k diagrams. How would you make an intrinsic GaAs sample n-type or p-type? What happens when GaAs is doped with Si? What is the nature of bonding in GaAs?

(b) An n-type Ge sample is 2 mm wide and 0.2 mm thick. A current of 10mA is passed through the sample (x-direction) and a magnetic field of 0.1 Weber/m² is directed perpendicular to the current flow (z-direction). The developed Hall voltage is 1.0mV. Calculate the Hall coefficient and electron concentration.

(c) (i) How do you represent analog colour TV signal mathematically?
(ii) How does discrete – time signal express in mathematics?
(iii) Determine whether the following discrete time signal is time invariant or not: \( y(n) = x(n) - x(n-2) \)
(iv) Represent recursive and non-recursive discrete-time system with the help of constant coefficient difference equation.
(v) What are the advantages of state-variable approach to analysis of a circuit (system)?
(vi) The continuous–time system as in Figure 1 consists of two integrator and two scalar multipliers. Write a differential equation that relates the output \( y(t) \) and input \( x(t) \).

(d) For the given circuit determine I by applying Superposition theorem.
(e) A material having $\sigma = 5.0$ s/m and $\varepsilon = 1$ is subjected to an electric field intensity of $E = 250 \sin 10^6 t$ (V/m). Find $J_D$ and $J_C$. Determine the frequency at which $J_D$ and $J_C$ are equal in magnitude.

(f) (i) Explain CMOS as an optical sensor.
(ii) How is it superior to a CCD?

2. (a) Derive one-dimensional continuity equation for holes in a semiconductor.
Reduce this expression to the standard diffusion equation by assuming that drift is negligible and there is no generation in the region.

(b) Obtain expression for short circuit current and open circuit voltage in an illuminated p-n junction. Explain how an illuminated p-n junction can be used as a photo detector or a photo-cell.

(c) Sketch the energy-band diagram of an ideal MOS capacitor at equilibrium. Explain with energy band diagram the following modes of operation of a MOS capacitor.
(i) Accumulation
(ii) Depletion
(iii) Inversion

3. (a) An abrupt Si p-n junction has
$N_A = 10^{21}$ / m$^3$ on p-side and
$N_D = 10^{21}$ / m$^3$ on n-side
Calculate the value of the contact potential and the total width of the depletion region under unbiased condition at 300 K. Derive the relations used in the computation. The intrinsic carrier concentration of Si is $1.5 \times 10^{10}$/m$^3$ and relative permittivity is 11.8.

(b) The bandgap of GaAs and AlAs are 1.43 eV and 2.16 eV respectively.
Assuming the bandgap of $Al_xGa_{1-x}$As to vary linearly with x between the two extreme values, find the value of x that would result in the emission of 680 nm from $Al_xGa_{1-x}$As.

(c) Obtain an expression for the drain current ($I_D$) as a function of drain voltage ($V_D$) for an n-channel MOSFET. Hence find the expression for the transconductance of the device in the saturation region.

4. Given the following state-space description of a system:
\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2
\end{bmatrix} =
\begin{bmatrix}
-3 & 1 \\
-2 & 0
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix} +
\begin{bmatrix}
0 \\
1
\end{bmatrix} u
\]
and
\[
y =
\begin{bmatrix}
1 & 0
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix}
\]
Where \( x_1, x_2 \) are the state – variable, \( u \) is the input and \( y \) is the output of the system:

(i) Find the state-transition matrix,
(ii) Transfer function of the system
(iii) Complete solution of the system when input is the step function and having zero initial condition.

(b) (i) Draw the block diagram of the system represented by the following constant coefficient difference equation:
\[
y(n) = b_x x(n) + b_y x(n-1) + ay(n-1)
\]
Where \( x(n) \) is the input sequence and \( y(n) \) is the output sequence

(ii) Digital Filter is a discrete-time system which is operated on an input sequence to produce an output sequence according to some computational algorithm.
\[
y(n) = \sum_{k=0}^{N} a_k x(n-k) + \sum_{k=1}^{N} b_k y(n-k)
\]
Find its transfer function.

(iii) Solve the difference equation using the one-sided z-transform
\[
y(n) = x(n) + by(n-1)
\]
With initial condition \( y(1) = P \)
Let the input be \( x(n) = e^{jwn} u(n) \)

(c) (i) Consider the system shown in the Figure below. Determine its impulse response of the system.

(ii) (a) Obtain the frequency response of the first-order recursive system represented by the difference equation
\[
y(n) = ay(n-1) + x(n)
\]
Considering the system is initially relaxed.
(b) Also plot magnitude and phase response

(iii) \( X(e^{j\omega}) = \frac{1}{(1-be^{-j\omega})(1-ae^{j\omega})} \)
Find the inverse Fourier Transform of above expression

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5. (a) (i) Determine the Thevenin and Norton equivalent of the circuit at the terminals X-Y

(ii) What load resistance connecting X and Y will draw maximum power? What is that power?

(b) A current source drives a parallel R- L-C network. If \( R = 6 \Omega \) and \( L = 3 \text{H} \) and \( C = \frac{1}{12} \text{F} \), determine its network function \( H(s) = \frac{V_c(s)}{I_s(s)} \), where \( i_s \) is the current source.

(c) (i) Consider the circuit shown below. Find the value of \( R_L \) and the Power delivered to each load resistor i.e. \( R_L \), \( R_L/2 \) and \( R_L/3 \) for maximum power transfer to the three resistor load.

(ii) Write the incidence matrix \( \mathbf{A} \) for the graph of Figure shown below.
6. (a) Two voltages of equal frequencies and amplitudes travel in opposite directions on a lossless transmission line. Find the expression of the total voltage pattern as a function of time and position on the line. Draw a neat sketch of the voltage wave pattern varying along Z-direction. Give the positions of voltage nulls.

(b) The electric field \( \vec{E} \) and the magnetic field \( \vec{H} \) in a source-free, homogeneous, isotropic region are given as
\[
\vec{E} = 100 (j\hat{x} + 2\hat{y} - j\hat{z})e^{j\omega t}
\]
\[
\vec{H} = (-\hat{x} + j\hat{y} - \hat{z})e^{j\omega t}
\]
Estimate the average power flow density and its direction in the region.

(c) A vertical Hertz dipole radiates 1 KW power. Find the electric field and the Poynting vector at a distance 10 km from the dipole in a horizontal plane passing through the dipole. What is the direction of electric field at the point?

7. (a) (i) Define gauge factor for a strain gage and derive an expression for it.

(ii) Explain the significance of piezo-resistivity in the expression.

(iii) A strain gage is used as a force measuring device as shown in the figure. All elements are strain gages with resistance 120\( \Omega \). Only \( R_1 \) is active. If the maximum gage current is 25 mA, determine the battery voltage \( E \). A calibrating resistance of 1.2 M\( \Omega \) is connected in parallel to \( R_1 \) and the trace shift in the CRO is 5 cm. Find the trace shift when the applied force causes 10 microstrain. If the temperature coefficient of the gage is \( 5 \times 10^4 \, \Omega/\Omega^\circ C \) what will be the output when the temperature compensation is not used?
(b) With a block diagram, explain the function of a dual slope DVM.

A dual slope integrating type of A/D converter has an integrating capacitor 0.1\(\mu\)F and a resistance of 100k\(\Omega\). The reference voltage is 2 V and the output of the integrator is not to exceed 10V. What is the maximum time required for the output voltage to be integrated?